UDC 004.7:654.195.6
DOI: 10.20535/2523-4455.2017.22.5.99947

Estimation of probabilistic processes in wireless networks of 802.11 standard

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***Abstract*—The article analyzes mechanisms of collision in a wireless 802.11 network with competitive access to a radio channel. The calculated relationships for determining the probabilities of collisions in the network as a whole in the presence of N active stations with a saturated load, and also for one station when implementing the binary exponential mechanism of increasing the competitive window are presented. The graphs, which illustrate the proposed equations, are given in this article too. An equation is also proposed for determining the probability of successful transmission of a data frame depending on the number of active stations and the initial value of the competitive window. The equations are obtained using the concept of a virtual competitive window.**

It was shown that in the case, where the number of active stations is one third of the contention window, the probability of collisions is about 0.25. This means that in the network with such number of active stations, each station will experience the collision at every fourth attempt to transmit a frame of data in average. The next attempt to transmit frame of data will be carried out by using the larger contention window. Collisions will cause the increase of transmitting time duration and its irregularity.

It was analyzed the efficacy of application of binary exponential law for the change of the contention window for active stations, caught in a conflict when trying to access the channel.

The equation for estimate the changes in the probability of collision and the number of competing stations in the access cycle was obtained by taken into account that it will take place when a quasi-stationary mode of operation of wireless networks is established.

It was shown that the most effectively this mechanism is triggered in the early stages of repeated access to radio channel. Those stations that carry out the 5-th and 6-th attempts to access channel have a little impact on the probability of collisions on the network. Such result may be explained because every station after successful transmit the data frame will begin a new attempt from the initial meaning of contention window. So, if the initial number of active stations in the wireless network approximately equal to the value of contention window, in steady state of saturated network the effective number of active stations will be twice less due to application of binary exponential law for the change of the competitive window. If the initial number of active stations will be only 30% from the value of contention window, the effective number of active stations will be less twice too.

It was gave the estimation of the overall probability of collisions in a saturated 802.11 network with competitive access to the radio channel. The threat of collisions increases quickly in comparison with the increase of number of active stations in the network

From the analysis of the received equations and graphs it follows that for the normal operation of the wireless network, it is necessary that the number of active stations that simultaneously compete for access to the radio channel does not exceed a third of the initial value of the contention window used in this network. In this case, the probability of successful transmission will be more than 0.85.

Ref. 10, fig. 7.

Keywords — wireless network; collision; contention window; probability.

I. INTRODUCTION

The basic technology of access to the radio channel in the networks of all specifications of the 802.11 standard is the technology of competitive access, which can be implemented using the distributed coordination function (DCF) directly or using the RTS/CTS algorithm, in which each node of the network, before sending data "On the air", first sends a special short message called RTS (Ready To Send) and indicates the readiness of this node to send data [1].

This function is based on the shared access method with carrier detection and the CSMA/CA collision avoidance mechanism.

This RTS-message contains information about the duration of the forthcoming transfer and about the addressee and is available to all nodes on the network (unless they are hidden from the sender). This allows other nodes to delay the transfer for a time equal to the declared duration of the message. The receiving station, having received the RTS signal, responds by sending a signal CTS (Clear To Send), indicating the readiness of the station to receive information. After this, the transmitting station sends a data packet, and the receiving station must transmit an ACK confirmation frame (Acknowledgment), confirming an error-free reception.

The RTS/CTS algorithm was significantly improved for use in networks operating under the specifications of 802.11n and ac, but the basic principle when sending an RTS frame – the principle of competitive access to the environment remained [1-4].

II. THE PROBLEM DEFINITION

Although the later versions of the 802.11 standard use additional mechanisms designed to reduce the likelihood of collisions in a network with a large number of active stations, the threat of collisions remains, and in the context of widespread wireless networks, it seems inevitable. In the case of 802.11n or 802.11ac networks that contain some nodes which conform to earlier specifications, or in the case of overlapping radio coverage areas for 802.11n/ac access points with radio coverage areas for 802.11g/a access points, collisions may occur due to overlap and adaptation of frequency bands as a result of the transition of 802.11n/ac access points to the modes corresponding to the 802.11g / a specifications [4, 5].

Virtually all users of 802.11 wireless networks are faced with such phenomena as disconnection, partial loss of information, a significant decrease in traffic intensity when receiving or transmitting audiovisual information in networks with several active subscribers. In wireless networks, this phenomenon is mainly due to the occurrence of collisions for a given station or the prolonged occupation of the radio channel as a result of collisions in which other stations of this network participate [1, 5-7].

The questions of determining the probability of collisions and the probability of successful transfer of data frames were considered in many works and were generalized, for example, in [1, 8, 9]. All authors note the complexity of the solution of such problems and apply different approaches and analytical dependencies that allow to make approximate estimates and assess the possibility of collision depending on the presence of various factors. Quantitative estimates made using the well-known relations tend to give a large error when comparing the calculated results with the results of field experiments [6].

The purpose of our investigation is to analyse the process of collisions by using the concept of virtual contention window and to propose some equations for modelling processes in wireless networks with competitive access.

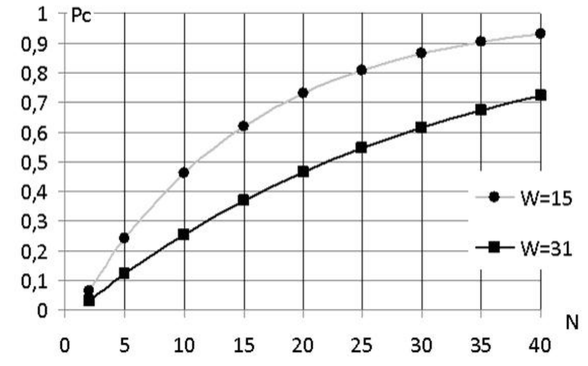

Fig. 1 Collision probability for one active station in a wireless network with N active stations in the saturated load mode.

III. THE ANALYSES OF COLLISION PROBABILITY

The initial data for the analysis are the minimum value of the contention window $CW_{\min} = W$ and the number of active stations in the network *N*. We also assume that the network uses a binary exponential function to change the value of the competitive window for stations participating in the collision [1]. The number of repeated attempts to access the channel in case of a collision is limited to the number *R*.

First, consider the collision probability for one network station, in addition to which there are also an (N-1) active stations, which simultaneously seek access to the channel for transmission.

The estimation will be carried out in accordance with the approach proposed in [10] for determining the virtual contention window. We consider the process of functioning of a network with a saturated load, as a certain quasi-stationary process. The basic idea of the concept is that in a stationary mode, the process of transferring data frames by each station of a network with a saturated load can be represented as occurring after the same time intervals due to using of a constant virtual contention window (*VCW*).

If there are only two active stations in the network, the probability p_{c1} that the given station will fall into a collision during the first stage of access to the channel when the current frame is transmitted will be equal to $p_{c1} = 1/W$, and the probability that the collision will not occur will be equal to $(1 - p_{c1})$.

The probability of no collision for a given station in the presence of (*N*-1) active stations in the network will be equal to $(1 - p_{c1})^{N-1}$, and the probability of collision for this station will be

$$p_c = 1 - (1 - p_{c1})^{N-1}. \qquad (1)$$

In this case, this p_{c1} is the probability of collision with only one active station of the wireless network.

The graph of this dependence for two values of the contention window is shown in Fig.1. These graphs show that the probability of collisions in wireless networks with saturated load is quite high. So, in the case, where the number of active stations is one third of the contention window, the probability of collisions is about 0.25. This means that in the network with such number of active stations, each station will experience the collision at every fourth attempt to transmit a frame of data in average. The next attempt to transmit frame of data will be carried out by using the larger contention window. Collisions will cause the increase of transmitting time duration and its irregularity.

Let's consider now the probability of successful transmission of a frame, which is only due to the procedure of competitive access to the channel, without taking into account the possibility of the cancellation of the frame due to the long waiting for access to the channel. Since in the event of a collision the station will make a second attempt (all *R* attempts) using an enlarged contention window, we will look for the probability of successful transmission p_s as the sum of the probabilities of successful transmission during all possible successive attempts. In the case of a quasi-stationary process in a wireless network, the likelihood of collision at all access stages will be the same and equal to p_c .

We will look for the probability of a successful transmission of data frame as the total probability of transmitting it by using of all (R) attempts.

The probability of successful transmission during the first attempt (in the first stage) will be determined by the relation

$$p_{s1} = \tau_1(1-p_c)\,,$$

where τ_1 is the probability of access to the channel in the first stage; $(1-p_c)$ – the probability that the collision at the first stage did not occur.

The probability of successful transmission in the second stage will be determined by the relation

$$p_{s2} = p_c\tau_2(1-p_c)\,,$$

where τ_2 is the probability of access to the channel in the second stage; $p_c(1-p_c)$ – the probability that the collision occurred in the first stage and did not occur in the second stage.

The probability of successful transmission during subsequent attempts will be determined by the ratio

$$p_{si} = p_c^{(i-1)}\tau_i(1-p_c)\,,$$

where τ_i is the probability of access to the channel at the i-th stage; $p_c^{(i-1)}(1-p_c)$ – the probability that the collision occurred at the previous (*i*-1) stages, and at the *i*-th stage a successful transfer occurred.

At the first stage the probability of access to the channel for all stations is the same and it is equal to one – each station loads the countdown counter from the same set of numbers and after the counter reaching the value "0" will get access to the channel. In the second stage, the value of the competitive window for the station that participated in the collision will double, and this in turn means that for it the probability of access to the channel will be half that of stations that did not participate in the collision. The change of the value of the contention window will be taken into account as a change in the probability of access to the wireless channel.

Based on the above remarks, we determine the total probability of successful transfer of one frame as

$$\begin{aligned} & p_s = \tau_1(1-p_c) + p_c\tau_2(1-p_c) + ... \\ & + p_c^{(i-1)}\tau_i(1-p_c) = \\ & = 1\cdot(1-p_c) + p_c\cdot\frac{1}{2}\cdot(1-p_c) + ... \\ & + p_c^{(R-1)}\cdot\frac{1}{2^{(R-1)}}\cdot(1-p_c) = \\ & = (1-p_c)\sum_{i=1}^{R}\left(\frac{p_c}{2}\right)^{i-1} = \frac{(1-p_c)[1-({}^{p_c}\!/_2)^R]}{(1-{}^{p_c}\!/_2)} \end{aligned} \quad , \quad (2)$$

where p_c is the collision probability for one active station, determined using (1).

The graph of the dependence (2) is shown in Fig.2. From these graphs it follows that if in the real network the conditions will be the same as in the network with saturated load, this network will cease normal operation even if the number of active stations will be not very large Assuming that the acceptable value of probability transmit data frame is 0.9, in the network with saturated load and with contention window W = 15, this probability can be achieved if the number of active stations is not more than 3 and in the network with contention window W=31 - no more than 7 active stations. It should be noted that the mode of the network, in which all stations will transmit full of stress is the worst mode of operation. This mode can only occur when all stations will transmit streaming information. For example, all stations will transmit video in real time.

Let us now consider how the probability of collision for one station will change when implementing a mechanism of increasing the competitive window in the network for stations that have collision.

Let's consider some abstract scenario in which all *N* active stations simultaneously start competing for access to the radio channel for transmission of their data frames. At the first access stage, all stations load countdown counters using the minimum value of the competitive window *W*.

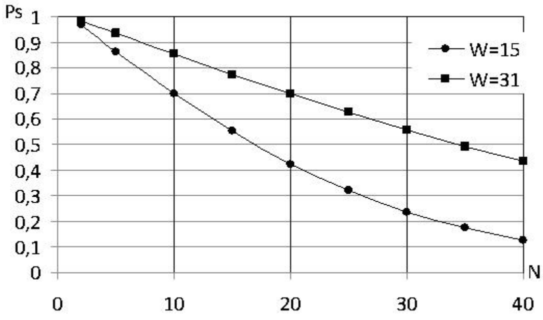

Fig. 2 Graphs of the probability of successful transmission of a frame p_s in a wireless network with different number of active stations *N*

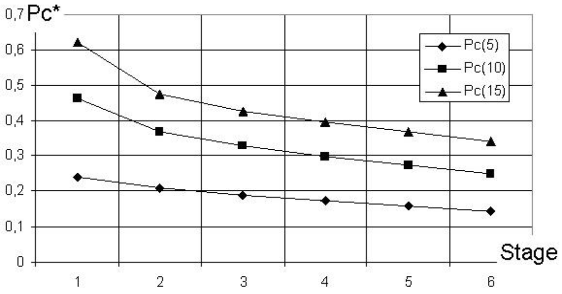

Fig. 3 Graphs of the change in the probability of collisions on different stages of access, for the total number of stations $N = 5$, 10, 15 and $W = 15$

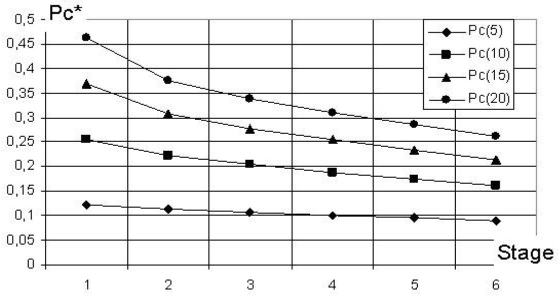

Fig. 4 Graphs of the change in the probability of collisions on different stages of access, for the total number of stations $N = 5$, 10, 15, 20 and $W = 31$

As it was said before the probability that the station did not fall into a collision is equal to $(1-p_{c1})$, then after the first attempt within some cycle of access at the first stage in the network will remain $N\cdot(1-p_{c1})$ stations, and $N\cdot p_{c1}$ stations will go to the second stage.

But for a station that has collision, the competitive window for the next access attempt is doubled. This leads to the fact that each of these stations will take part in the competition for access to the radio channel two times less often than stations that did not fall into a collision and stayed in the first access stage.

The stations that have collision and moved to the third stage of access to the radio channel will take part in the competition for access two times less often as the stations that are in the second stage of access to the channel.

The change in the size of the competitive window leads to the fact that during a single conditional cycle corresponding to the contention window, a smaller number of stations will participate in the competition for access to the channel. This in turn, in accordance with (1), will reduce the probability of collisions for all competing stations.

Let's consider how the probability of collision and the number of active stations participating in the competition for access to the channel during a certain conditional cycle of access, which in a quasi-stationary mode is a virtual competitive window, will change.

In the first access stage, the probability of collision for each station is determined by the relation (1). The number of stations that will participate in the competition in the second access cycle to the channel, in accordance with the previously made comments, will be determined by the ratio

$$N_2^* = N\cdot(1-p_{c1}) + \frac{1}{2}p_{c1}\cdot N\,. \qquad (3)$$

During the second access cycle to the channel, the probability of collision for each station is determined by the relation

$$p_{c2} = 1-(1-p_{c1})^{(N_2^*-1)}\,. \qquad (4)$$

The number of stations that will participate in the competition in the third cycle of channel access will be equal to

$$\begin{aligned} N_3^* &= N_2^*\cdot(1-p_{c2}) + \\ &+\frac{1}{2}p_{c2}\cdot N_2^* + \frac{1}{2}\cdot p_{c2}^2\cdot N_2^*\,. \end{aligned} \qquad (5)$$

Changes in the probability of collision and the number of competing stations in the access cycle will take place throughout the time, after which a quasi-stationary mode of operation of wireless networks is established. In the R-th access cycle, the probability of collision and the number of competing stations will be determined by the relations

$$p_{cR} = 1-(1-p_{c1})^{(N_R^*-1)}\,, \qquad (6)$$

$$\begin{aligned} N_R^* &= N^*_{(R-1)}(1-p_{(R-1)}) + \frac{1}{2}p_{(R-1)}N^*_{(R-1)} + \\ &+\frac{1}{2}p^3_{(R-1)}N^*_{(R-1)} + \ldots + \frac{1}{2}p^{(R-1)}_{(R-1)}N^*_{(R-1)} \end{aligned}\,. \qquad (7)$$

Graphs for the change in the probability of collisions in a wireless network with a binary exponential change in the value of the competitive window, for stations that are in collision are shown in Fig. 3 and 4, And the graphs of the number of active stations participating in the competition for access to the channel are shown in Fig. 5 and 6.

These graphics depending enable to assess the efficacy of the mechanism binary exponential law for the change of the contention window for active stations, caught in a conflict when trying to access the channel. As can be seen from Fig. 3 and Fig. 4, the most effective this mechanism is triggered in the early stages of repeated access to radio channel. Those stations that carry out the 5-th and 6-th attempts to access channel have a little impact on the probability of collisions on the network. Such result may be explained because every station after successful transmit the data frame will begin a new attempt from the initial meaning of contention window. So, if the initial number of active stations in the wireless network approximately equal to the value of contention window, in steady state of saturated network the effective number of active stations will be twice less due to application of binary exponential law for the change of the competitive window. If the initial number of active stations will be only 30% from the value of contention window, the effective number of active stations will be less twice too. Based on analysis of these diagrams Fig. 3 – Fig. 6 it can be concluded that the increased number of repeated attempts to stations that have experienced several consecutive collisions is impractical because it almost don't lead to changes in the operation of the wireless network.

Now we will estimate the overall probability of collisions in a saturated 802.11 network [1] with competitive access to the radio channel. In this case it is a question of collisions between any stations of the network, which leads to unproductive time expenditures. We consider that all active stations of the network are in the first stage of access to the radio channel and use the number from the set {1,2, ..., *W*} to load the countdown counter.

In our case, the minimum value of the contention window is marked as $CW_{\min} = W$. The number of active stations *N*. We will find the overall probability of collisions by using the relations known from combinatorics for the arrangement. The probability of collision is found as the ratio of possible arrangement of size *m* from *n*, in which there are obligatorily repetitions of numbers loaded into the counters of different stations to the arrangement with unlimited repetitions of size *m* from *n*.

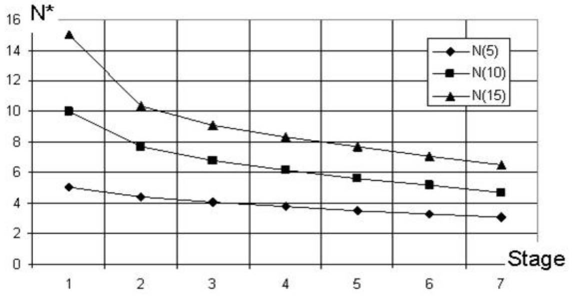

Fig. 5 Graphs of the change in the number of active stations *N** competing for access to the channel on different stages of access, for the total number of stations *N* = 5, 10, 15, 20 and *W* = 15

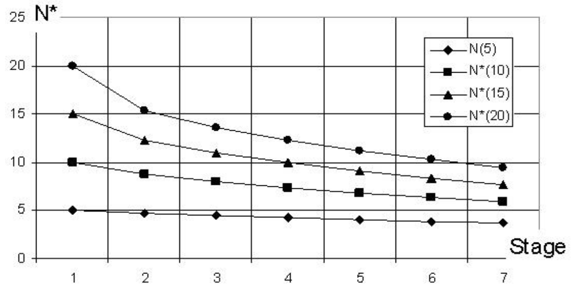

Fig. 6 Graphs of the change in the number of active stations *N** competing for access to the channel on different stages of access, for the total number of stations *N* = 5, 10, 15, 20 and *W* = 31

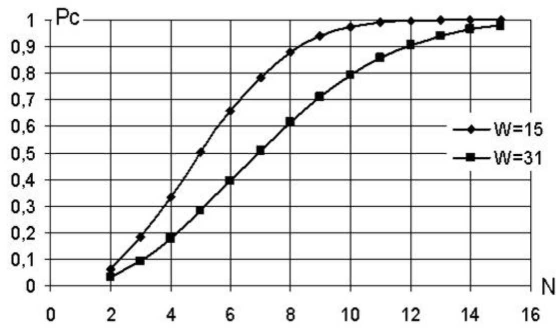

Fig. 7 Graph of the probability P_c of overall collision in a wireless network with *N* active stations for two meanings of contention window *W*

The arrangement with unlimited repetitions of size *m* from *n* is determined by the ratio $\tilde{A}_n^k = n^k$. The arrangement of size *m* from *n* without repeats is determined by the dependence $A_n^k = \frac{n!}{(n-k)!}$. So, the required overall probability of collisions is determined as

$$P_c = \frac{\tilde{A}_n^k - A_n^k}{\tilde{A}_n^k} = 1 - \frac{W!}{(W-N)!W^N}. \qquad (8)$$

Graph of the relation (8) is shown in Fig. 7. These graphs shows that active stations of these networks with a large number of subscribers have a big risk of collisions. The threat of collisions increases quickly in comparison with the increase of number of active stations in the network

The obtained equation gives the ability of more accurately take into account the amount of time caused by collisions in a wireless network with competitive access to radio channel and to estimate the characteristics of segment wireless network as a whole if there is a large number of customers in this segment.

CONCLUSIONS

1) Calculation relationships were obtained to determine the probabilities of collision and successful transmission of frames on a wireless network with saturated upload and competitive access to the channel based on the concept of a virtual contention window.
2) From the analysis of the received equations and graphs it follows that for the normal operation of the wireless network, it is necessary that the number of active stations that simultaneously compete for access to the radio channel does not exceed one-third of the initial value of the contention window used in this network. In this case, the probability of successful transmission will be more than 0.85.
3) In a network with a saturated load of active stations, there is a high probability of collisions even when implementing the mechanism of repeated attempts to access the radio channel using a competitive window enlarged in binary power law. If there are 15 active stations and the value of the

competitive window $W = 15$ and R=6, the probability of collision for each station is 0.34, and in the case of W=31, this probability is 0.26.

4) In the case of a large number of active users of a wireless network, it is advisable to use a larger number of access points with a smaller radius of action to reduce the number of active subscribers in one segment of the network.

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Надійшла до редакції 24 квітня 2017 р.

УДК 004.7:654.195.6

Оцінювання ймовірнісних процесів у безпроводових мережах стандарту 802.11

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***Реферат*—У статті проаналізовано механізми виникнення колізій у безпроводових мережах 802.11 з конкурентним доступом до радіоканалу. Запропоновано розрахункові співвідношення для визначення ймовірностей колізій в мережі в цілому за наявності N активних станцій з насиченим навантаженням, а також для однієї станції за умови реалізації двійкового показникового механізму збільшення конкурентного вікна. Наведено графіки, що ілюструють отримані аналітичні співвідношення. Запропоновано також співвідношення для визначення ймовірності успішного передавання кадру даних залежно від кількості активних станцій і початкового значення конкурентного вікна. Співвідношення отримані з використанням концепції віртуального конкурентного вікна.**

Бібл. 10, рис. 7.

Ключові слова — безпроводова мережа; колізія; конкурентна вікно; ймовірність.

УДК 004.7:654.195.6

Оценка вероятностных процессов в беспроводных сетях стандарта 802.11

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***Реферат*—В статье проанализированы механизмы возникновения коллизий в беспроводных сетях 802.11 с конкурентным доступом к радиоканалу. Предложены расчетные соотношения для определения вероятностей коллизий в сети в целом при наличии N активных станций с насыщенной нагрузкой, а также для одной станции при реализации двоичного показательного механизма увеличения конкурентного окна. Приведены графики, иллюстрирующие полученные аналитические соотношения. Предложено также соотношение для определения вероятности успешной передачи кадра данных в зависимости от количества активных станций и начального значения конкурентного окна. Соотношения получены с использованием концепции виртуального конкурентного окна.**

Библ. 10, рис. 7.

Ключевые слова — беспроводная сеть; коллизия; конкурентное окно; вероятность.