Електронні системи та сигнали

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Apparent Power and Efficiency of Three-Phase Four-Wire System

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***Abstract*—The differences between Buchholz's formula of apparent power and proposed formulation were demonstrated in the case of three-phase four-wire power system. It was shown that voltage integral from proposed formula of apparent power is the value of power losses in active resistances of power cable, which is due to load line-to-neutral voltages and short circuit mode of load. A new physical sense of apparent power was established, it is a geometric mean value of short-circuit power and power losses, caused by line currents. This definition of apparent power of three-phase power supply system is completely equivalent to definition standardized by IEEE. Conditions to achieve unit power factor and maximum efficiency of three-phase four-wire power system were established. The formula for determining efficiency of three-phase four-wire power system for unbalanced non-sinusoidal mode with a given load power factor and the known load factor has been derived. Computer simulation in MATLAB Simulink showed good agreement between the experimental data and the theoretical curves for efficiency both in the presence of the SAF, or without it.**

Ref. 8, Fig. 5, Tab. 1.

Keywords* — *apparent power; power factor; efficiency; control strategy; shunt active filter.

I. Introduction

The concept of Fryze's active current [1] as the vector of line currents with minimum mean-square norm, which provides the same load power at the given line-to-neutral voltages is widely applied to minimize the effective value of line current vector and associated power losses in power cable by using of shunt active filter (SAF) [2], [3]. The function of SAF is to generate the line current inactive components that non-linear load requires, then active current will be consumed from the source with minimum losses, unit power factor and maximum efficiency. However, in determining of power factor the apparent power appears that is calculated in number of papers [2], [3] by the Buchholz's formula as product of rms values of line currents and line-to-neutral voltages. The independence of the apparent power in Buchholz's formula from the ratio of cable resistances is doubtful that it is correct for four-wire systems with non-zero neutral current. Besides, only a few studies devoted to the direct calculation of power efficiency for selected modes of three-phase four-wire supply system even under simplified resistive models of the power cable have been reported [4], [5]. Therefore, studies aimed at further development of the power theory in terms of finding ratios for apparent power, the power factor and the effect of load power factor to the efficiency of three-phase four-wire system with SAF are relevant.

II. New Physical Sense of Apparent Power

The correct definition of apparent power is crucial for the building of energy-efficient means of active filtering as a perfect power delivery mode is characterized by the equality of the apparent power to the active one. The ratio of active to apparent power is called the power factor

$$\lambda = P_L / S = \frac{1}{\tau}\int_0^{\tau} \mathbf{u}^T(t)\mathbf{i}(t)dt / S. \qquad (1)$$

where P_L is the load active power; S – apparent power; $\mathbf{u}(t)$, $\mathbf{i}(t)$ – three-coordinate vectors of instantaneous values of line-to-neutral voltages and line currents; τ is the period of the network voltage; ^{T} – sign of transposition. In recent works [2], [3] for apparent power as three-wire and four-wire of three-phase power systems a Buchholz's formula is applied as the product of rms norms of the vectors of the phase voltages and line currents:

$$S = \sqrt{\frac{1}{\tau}\int_0^\tau \mathbf{i}^T(t)\mathbf{i}(t)dt\,\frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{u}(t)dt}. \qquad (2)$$

However, the neutral wire, in which the sum current of the line wires flows, is a significant difference of the four-wire system compared with the three-wire system, since power losses increase in this case. As was shown in paper [4], [5] the independence of the apparent power in Buchholz's formula from the ratio of the cable resistances r, r_N (see Fig. 1) is not correct for four-wire systems with non-zero neutral current.

Correctly apparent power of three-phase four-power system is defined as the maximum value of active load power that can be achieved under given voltages $\mathbf{u}(t)$ and limited energy losses ΔP in active resistances of power cable during the period of the network voltage [6]. As a result of solving this extreme problem the expression for the optimal linear current vector was obtained [7]:

$$\mathbf{i}_{opt}(t) = \sqrt{\frac{\Delta P}{\frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{R}^{-1}\mathbf{u}(t)dt}}\,\mathbf{R}^{-1}\mathbf{u}(t) =$$

$$= \sqrt{\frac{\Delta P / r}{\frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{u}_\sigma(t)dt}}\,\mathbf{u}_\sigma(t), \qquad (3)$$

where $\mathbf{R} = r\mathbf{E} + r_N\mathbf{j}\mathbf{j}^T$ is matrix of resistance losses in the power cable of three-phase four-wire power system that is symmetrical about the main diagonal; r is the resistance of each phase wire, r_N is the resistance of the neutral wire; $\mathbf{j}^T = \|1 \;\; 1 \;\; 1\|$; $\mathbf{u}_\sigma(t) = r\mathbf{R}^{-1}\mathbf{u}(t) = [\mathbf{E} - (\sigma/3)\mathbf{j}\mathbf{j}^T]\mathbf{u}(t)$; $\mathbf{E}$ is identity matrix; $\sigma = 3r_N/(r + 3r_N)$ is optimum attenuation coefficient of zero-sequence component of the instantaneous line-to-neutral voltage vector, that minimizes energy losses in the power cable of the four-wire three-phase network with asymmetric non-sinusoidal voltages.

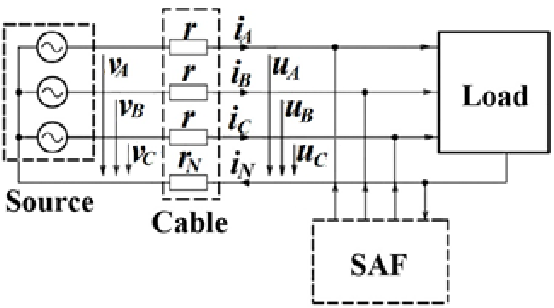

Fig. 1. The model of three-phase four-wire power system with SAF

In view of (3) a new formula for determining the apparent power of three-phase four-wire power system is given by [7]

$$S = P_{max} = \frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{i}_{opt}(t)dt =$$

$$= \sqrt{\frac{1}{\tau}\int_0^\tau \mathbf{i}^T(t)\mathbf{R}\mathbf{i}(t)dt\,\frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{R}^{-1}\mathbf{u}(t)dt}. \qquad (4)$$

The apparent power of three-phase three-wire system is described by (4) too and corresponds to the Buchholz's formula (2) because in this case $\mathbf{R} = r\mathbf{E}$. For four-wire system matrix R affects the integral expressions of (4) in the presence of zero sequence components of voltage and current vectors and apparent power formula can be represented as [7]

$$S = \sqrt{[U_\perp^2 + (1-\sigma)U_0^2][I_\perp^2 + (1-\sigma)^{-1}I_0^2]}, \qquad (5)$$

where $I_\perp^2 = \frac{1}{\tau}\int_0^\tau \mathbf{i}_\perp^T(t)\mathbf{i}_\perp(t)dt$; $I_0^2 = \frac{1}{\tau}\int_0^\tau \mathbf{i}_0^T(t)\mathbf{i}_0(t)]dt$;

$U_\perp^2 = \frac{1}{\tau}\int_0^\tau \mathbf{u}_\perp^T(t)\mathbf{u}_\perp(t)dt$; $U_0^2 = \frac{1}{\tau}\int_0^\tau \mathbf{u}_0^T(t)\mathbf{u}_0(t)]dt$ – squares of rms values of the vector's orthogonal components which are localized, respectively, in $\alpha\beta$-plane [2] (marked by sub-symbol $\perp$) and collinear to ort vector $\mathbf{j}/\sqrt{3}$ (zero sequence component, marked by sub-symbol 0). Unlike (2), it is clearly visible in (5) the dependence of the apparent power and, consequently, power factor from the ratio of the resistances, giving by the coefficient σ in the presence of the vector's zero sequence components.

In paper [8] was shown that proposed formulas of apparent power (4), (5) are fully equivalent to standardized one IEEE Std 1459-2010[1] and remove the uncertainty factor in that IEEE standard.

Let's find the value of active power in the circuit in Fig. 2, formed from the scheme of Fig. 1 by replacing of short circuit instead of load. Potential of common point of resistors denote $\varphi(t) = \frac{u_A(t) + u_B(t) + u_C(t)}{r(3/r + 1/r_N)}$, then

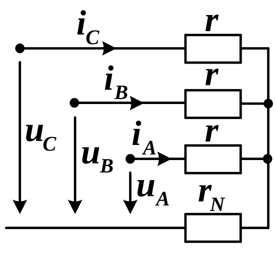

Fig. 2. Short-circuit scheme

[1] IEEE standart definitions for the measurements of electrical power quantities under sinusoidal, nonsinusoidal, balanced and nonbalanced conditions.

$$P_0 = \frac{1}{\tau}\int_0^\tau \begin{Vmatrix} u_A(t) & u_B(t) & u_C(t) \end{Vmatrix} \begin{Vmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \end{Vmatrix} dt =$$

$$= \frac{1}{r\tau}\int_0^\tau \begin{Vmatrix} u_A(t) & u_B(t) & u_C(t) \end{Vmatrix} \begin{Vmatrix} u_A(t)-\varphi(t) \\ u_B(t)-\varphi(t) \\ u_C(t)-\varphi(t) \end{Vmatrix} dt =$$

$$= \frac{1}{r\tau}\int_0^\tau \mathbf{u}^T(t)[\mathbf{u}(t) - \frac{u_A(t)+u_B(t)+u_C(t)}{r(3/r+1/r_N)}\mathbf{j}]dt = \quad (6)$$

$$= \frac{1}{r\tau}\int_0^\tau \mathbf{u}^T(t)[\mathbf{u}(t) - \frac{3}{3+r/r_N}\mathbf{u}_0(t)]dt =$$

$$= \frac{1}{r\tau}\int_0^\tau \mathbf{u}^T(t)[\mathbf{u}(t) - \sigma\mathbf{u}_0(t)]dt = \frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{R}^{-1}\mathbf{u}(t)dt.$$

Thus, the voltage integral from the expression (4) of apparent power is the value of power losses in active resistances of power cable, which is due to load line-to-neutral voltages and short circuit mode of three-phase load:

$$\frac{1}{\tau}\int_0^\tau \mathbf{u}^T(t)\mathbf{R}^{-1}\mathbf{u}(t)dt = P_0. \quad (7)$$

With this in mind the physical meaning of apparent power given by formula (4) formulated as a geometric mean value of voltage source short-circuit power P_0 and power losses ΔP, caused by line and neutral currents:

$$S = \sqrt{P_0 \Delta P}. \quad (8)$$

III. Conditions to Achieve Unit Power Factor and Maximum Efficiency of Three-Phase Four-Wire Power System

Whereas the definition of apparent power by the expression (5), power factor will be equal to the unit only if the line current vector is proportional to the vector of line-to-neutral voltages with the partial attenuation of zero-sequence component:

$$\mathbf{i}(t) = G\mathbf{u}_\sigma(t) = G[\mathbf{u}_\perp(t) + (1-\sigma)\mathbf{u}_0(t)], \quad (9)$$

where G – any real constant, since in this case in formula (1)

$$P_L = G[U_\perp^2 + (1-\sigma)U_0^2];$$

$$S = \sqrt{[U_\perp^2 + (1-\sigma)U_0^2]G^2[U_\perp^2 + (1-\sigma)U_0^2]}; \quad \lambda = 1.$$

If SAF introduced in power system, the three-phase source current vector that provides unit power factor in asymmetric non-sinusoidal mode, is given by [7]

$$\mathbf{i}_S(t) = G_\sigma \mathbf{u}_\sigma(t), \quad (10)$$

where the value of coefficient G_σ is found from the condition of zero energy consumption by SAF on period τ:

$$G_\sigma = \frac{P}{U_\perp^2 + (1-\sigma)U_0^2}. \quad (11)$$

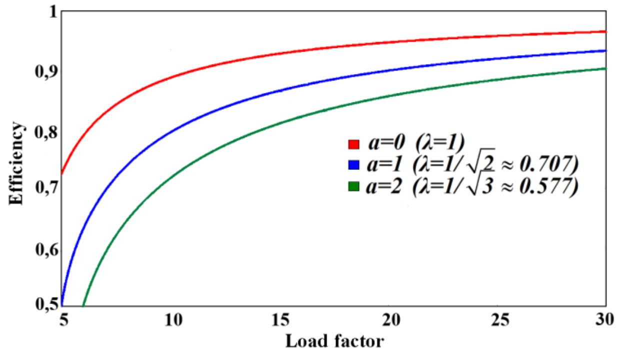

Fig. 3. The dependences of the efficiency from the load factor and power factor.

Vector correlation (10) with the meaning of coefficient by formula (11) specifies the value of the Fryze's active current vector in the case of three-phase four-wire network. Formulas (10), (11) identify the original SAF's control strategy [5]:

$$\mathbf{i}_{SAF}(t) = \mathbf{i}(t) - G_\sigma \mathbf{u}_\sigma(t), \quad (12)$$

that ensures minimum losses in power cables and maximum efficiency. In view of (10), (11) this minimum value is

$$\Delta P_{min} = \frac{1}{\tau}\int_0^\tau \mathbf{i}_S^T(t)\mathbf{R}\mathbf{i}_S(t)dt = \frac{P^2 r}{U_\perp^2 + (1-\sigma)U_0^2} = rG_\sigma P. \quad (13)$$

Under these minimum losses maximum efficiency of three-phase four-wire power system will be

$$\eta_{max} = \frac{P}{P+\Delta P_{\min}} = \frac{P}{1+\Delta P_{\min}/P} = \frac{1}{1+rG_\sigma}.$$

IV. The Effect of Power Factor to Efficiency of Three-Phase Four-Wire Power System and Its Verification

As a result of formula's (4) application the expression for efficiency of three-phase four-wire system $\eta = P_L/(P_L + \Delta P)$, was obtained in [5] as the solution of a quadratic equation

$$\eta^{-2} - \eta^{-1}k_L + k_L - 1 + 1/\lambda^2 = 0, \quad (14)$$

where $k_L = P_0/P_L$ specified as load factor; is the short-circuit active power; $\mathbf{v}(t)$ is vector of instantaneous values of input line-to-neutral voltages.

Under the condition $k_L \geq 2 + 2/\lambda$ power efficiency is defined from equation (14) as follows:

$$\eta = \frac{1}{0.5k_L - \sqrt{0.25k_L^2 - k_L - a}}, \quad (15)$$

where $\alpha = \lambda^{-2} - 1$.

Diagrams of the efficiency dependences from the load factor k_L at values of $a = 0, 1, 2$ are shown in Fig. 3. From its analysis it follows that with decreasing of load power factor the effectiveness of application of SAF increases.

To verify the formula (15) for efficiency the MATLAB Simulink model of three-phase four-wire power system with SAF and nonlinear load was built (see Fig 4).

It consists of three symmetric sinusoidal power sources A, B, C, resistances of the phase wires (r_A, r_B, r_C) and neutral wire (r_N). Non-linear load is presented by three-phase rectifier with rectifier diodes A, B, C and active impedance R_L. SAF is simulated by dependent current sources whose values are formed in accordance with the formula (12).

Active power of short-circuit is calculated according to the formula $P_0 = 2U^2 / r = 145200$ W, where U=220 V is the rms value of the phase voltage source, r = 1 Ω is the value of phase wires resistance.

Load power is determined by the formula [8]

$$P_L = \frac{U^2}{R_L}\left(1+\frac{3\sqrt{3}}{4\pi}\right),$$

where R_L is the value of the load impedance.

Load factor is

$$k_L = \frac{P_0}{P_L} = \frac{3R_L}{r\left(1+\frac{3\sqrt{3}}{4\pi}\right)} \approx 2.122 R_L / r.$$

The power factor's value of three-phase rectifier with resistive load is calculated using the formula [8]:

$$\lambda = \frac{\lambda_0}{\sqrt{1+r_N / r}},$$

where $\lambda_0 = \sqrt{\frac{1}{3}+\frac{\sqrt{3}}{4\pi}} \approx 0.686$ the power factor at the zero neutral conductor resistance.

The simulation results for different values of the load and neutral resistance are given in Table 1.

The theoretical dependences of efficiency from the power factor λ given by formula (15) for two values of load resistance and computer experiment data are shown in Fig. 5. Data for λ = 1 were obtained by connecting SAF.

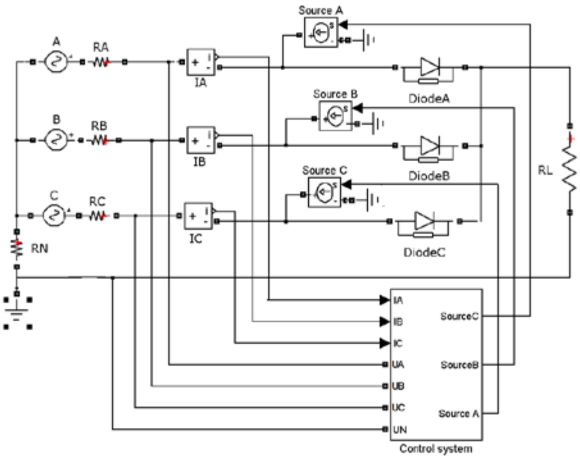

Fig. 4. MATLAB Simulink model of three-phase four-wire power system with SAF

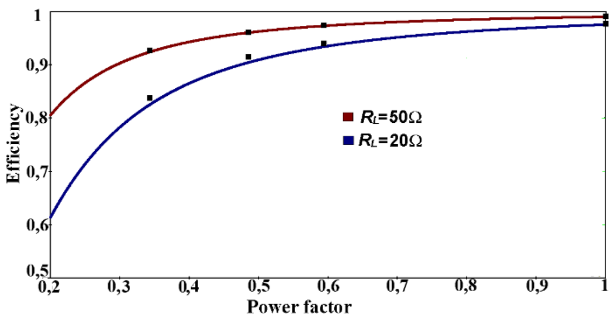

Fig. 5. The diagram of theoretical efficiency in accordance with the formula (15) (solid lines) and computer simulation results (* marked pots)

Generally computer simulation showed good agreement between the experimental data and the theoretical curves for efficiency both in the presence of the SAF, or without it.

CONCLUSION

1) It was shown that voltage integral from the formula (4) of apparent power is the value of power losses in active resistances of power cable, which is due to load line-to-neutral voltages and short circuit mode of load.
2) A new physical sense of apparent power was established, it is a geometric mean value of voltage source short-circuit power and power losses, caused by line and neutral currents. This definition of apparent power of three-phase power supply system is completely equivalent to definition standardized by IEEE.

TABLE 1. THE SIMULATION RESULTS

	r_N = 0.33Ω		r_N = 1Ω		r_N = 3Ω	
λ	0.594	1	0.485	1	0.343	1
r=1Ω; R_L= 20 Ω						
P_L	2998	3258	2818	3258	2369	3258
P_S	3188	3334	3079	3334	2827	3334
$\eta=P_L/P_S$	0.9403	0.977	0.915	0.977	0.838	0.977
r=1Ω; R_L= 50 Ω						
P_L	1294	1340	1260	1340	1170	1340
P_S	1329	1352	1310	1352	1261	1352
$\eta=P_L/P_S$	0.974	0.991	0.962	0.991	0.927	0.991

3) The formula for determining efficiency of three-phase four-wire power system for unbalanced non-sinusoidal mode with a given load power factor and the known load factor has been derived and verified by computer simulation.

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Повна потужність та коефіцієнт корисної дії трифазної чотирипровідної системи живлення

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***Реферат*—Несиметрія навантаження трифазних трипровідних систем живлення призводить до погіршення якості електричної енергії в точках загального під'єднання, що проявляється у появі пульсацій миттєвої потужності та додаткових втрат на опорах ліній електропередач, а також у несиметрії напруг живлення. Ефективна компенсація асиметрії навантаження можлива лише за допомогою силових активних фільтрів, при цьому паралельні активні фільтри характеризуються меншими втратами потужності, ніж послідовні. Функція паралельного активного фільтру полягає у тому, щоб генерувати у навантаження неактивні компоненти струму, що потребує нелінійне навантаження. Тоді від джерела буде споживатися активний струм з мінімальними втратами, одиничним значенням коефіцієнта потужності та максимальним значенням коефіцієнта корисної дії системи живлення. Проте у визначенні коефіцієнта потужності фігурує повна потужність, яка зазвичай розраховується за формулою Бухгольца у вигляді добутку середньоквадратичних значень лінійних струмів і фазних напруг. Незалежність повної потужності в формулі Бухгольца від співвідношення опорів лінійних проводів та нейтралі свідчить про її неадекватність для чотирипровідних систем з ненульовим нейтральним струмом.**

В роботі була продемонстрована різниця між формулою повної потужності Бухгольца та запропонованою формулюванням у випадку трифазної чотирипровідної системи живлення. Показано, що інтеграл напруг в запропонованій формулі повної потужності є величиною втрат потужності в активних опорах силового кабелю, обумовленою

фазними напругами джерела в режимі короткого замикання навантаження. Був встановлений новий фізичний зміст повної потужності: це геометричне середнє значення потужності короткого замикання трифазного джерела на силовий кабель і потужності втрат, викликаної лінійними струмами та струмом нейтралі. Таке визначення повної потужності трифазної системи живлення повністю еквівалентно стандарту IEEE. Знайдено умови досягнення одиничного значення коефіцієнта потужності та максимального значення коефіцієнта корисної дії трифазної чотирипровідної системи живлення з паралельним активним фільтром. Виведено формулу для визначення коефіцієнта корисної дії трифазної чотирипровідної системи живлення для незбалансованого несинусоїдного режиму з заданим коефіцієнтом потужності і відомим коефіцієнтом навантаження. Комп'ютерне моделювання в MATLAB Simulink показало хороший збіг експериментальних даних з теоретичними кривими коефіцієнта корисної дії як за наявності паралельного активного фільтра, так і без нього.

Бібл. 8, рис. 5, таб. 1.

Ключові слова — повна потужність; коефіцієнт потужності; ефективність; стратегія керування; паралельний активний фільтр.

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Полная мощность и коэффициент полезного действия трехфазной четырехпроводной системы питания

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***Реферат*—Показано, что интеграл напряжений в предложенной формуле полной мощности представляет собой величину мощности потерь в активных сопротивлениях силового кабеля, обусловленную фазными напряжениями источника в режиме короткого замыкания нагрузки. Был установлен новый физический смысл полной мощности: это среднее геометрическое значение мощности короткого замыкания трехфазного источника на силовой кабель и мощности потерь, вызванной линейными токами и током нейтрали. Это определение полной мощности трехфазной системы питания полностью эквивалентно стандарту IEEE. Выведена формула для определения коэффициента полезного действия трехфазной четырехпроводной системы питания для несбалансированного несинусоидального режима с заданным коэффициентом мощности и известным коэффициентом нагрузки. Компьютерное моделирование в MATLAB Simulink показало хорошее совпадение экспериментальных данных с теоретическими кривыми коэффициента полезного действия как при наличии параллельного активного фильтра, так и без него.**

Библ. 8, рис. 5, таб. 1.

Ключевые слова — полная мощность; коэффициент мощности; коэффициент полезного действия; параллельный активный фильтр