Електронні системи та сигнали

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Frequency Response Analysis of Buck-Boost Inverter

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Abstract—The paper deals with processes analysis in inverter circuits. These processes are described by differential equations with periodical coefficients. Since control signals and voltages operating in an inverter circuit are independent theirs frequencies are often incommensurable. In order to obtain steady-state periodic solutions ordinary differential equations with one independent time variable are expanded into partial differential equations with two independent variables of time. Obtained partial differential equations with periodical coefficients are transformed into equations with constant coefficients by using the Lyapunov transformation. Since these equations are defined in the domain of two time variables theirs solutions are determined by using the two dimensional Laplace transform. After solving differential equations a steady-state solution is given in the form of the double Fourier series. Frequency responses are defined as functions of Fourier coefficients dependent on control signal and power voltage frequencies. Results of calculations of frequency responses for a buck-boost inverter are presented.

Ref. 11, fig. 7.

Key words: expansion of differential equations, frequency responses, two-dimensional Laplace transform

I. INTRODUCTION

The analysis of transient and steady-state processes in periodic time-varying circuits a difficult task connected with necessity to take into account element models, control signals and power sources. For linear circuits there are different analysis methods among which it should be noted the frequency response method [1]. In periodic time-varying circuits this method can not be used since processes in such circuits are described by differential equations with periodically variable coefficients.

Processes in such circuits can be analysed by different method [2, 3, 4, 5] in case these circuits are controlled by signals with the same or proportional frequencies. For inverters that could be worked with incommensurable signal frequencies determining steady-state processes are a difficult problem. In the domain of one time variable one cannot find periodic steady-state behaviour.

The task of finding periodic steady-state behaviour can be realised by an expansion of ordinary differential equations with one time variable into partial differential equations with two time variables [6].

The aim of this article is to find steady-state processes in an inverter circuit worked with incommensurable signal frequencies and analyze frequency responses. The method is based on the expansion of differential equations and solving these equations by using the Lyapunov transformation and the two dimensional Laplace transform. Steady-state processes are represented in the form of the double Fourier series. An example of a buck-boost inverter is considered and a set of frequency responses is presented.

II. MATHEMATICAL MODEL

Let us consider the circuit of the buck-boost inverter presented in Fig. 1.

The switches S_1 and S_2 are ideal and switch alternately, if S_1 is closed, S_2 is open as shown in Fig. 2 (we consider that when $s(t)=1$ S_1 is closed and S_2 is open). The switching function $s(t)$ is periodic with the period Θ and has the duration equal to t_1.

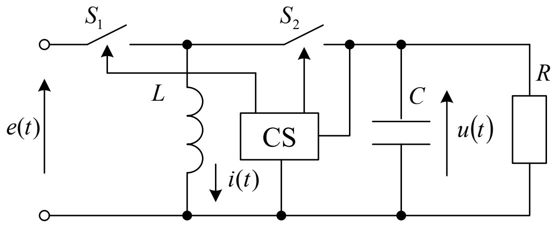

Fig. 1. Topology of the inverter

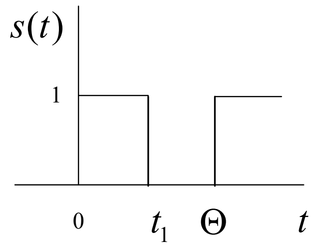

Fig. 2. Switching function

The topology of the inverter is changed periodically and the differential equations describing the processes in the circuit have the form

$$\frac{di(t)}{dt}=-\frac{R_L}{L}i(t)-\frac{1-s(t)}{L}u(t)+\frac{s(t)e(t)}{L},\qquad \frac{du(t)}{dt}=\frac{1-s(t)}{C}i(t)-\frac{1}{RC}u(t); \tag{1}$$

where R_L is the resistance of an inductor, $e(t)=e(t+T)$, T is the period of a power source.

In the matrix form set (1) takes the form

$$\frac{dX(t)}{dt}=A(t)X(t)+B(t),$$

where $X(t)=\begin{pmatrix} i(t)\\ u(t)\end{pmatrix}$ is a vector of state variables,

$$A(t)=\begin{pmatrix} -\frac{R_L}{L} & -\frac{1-s(t)}{L}\\ \frac{1-s(t)}{C} & -\frac{1}{RC}\end{pmatrix},\ B(t)=\begin{pmatrix} \frac{s(t)}{L}\\ 0\end{pmatrix}e(t).$$

In case of incommensurable periods of the source voltage $e(t)$ and the switching function $s(t)$ the steady-state behavior to (1) is not periodic.

Let us expand the domain of the differential equation (1) from one time variable t to two time variables t and τ as follows [6]

$$\frac{\partial X(t,\tau)}{\partial t}+\frac{\partial X(t,\tau)}{\partial \tau}=A(\tau)X(t,\tau)+B(t,\tau). \tag{2}$$

In this domain there exists a periodic steady-state process, i.e., $X_s(t,\tau)=X_s(t+T,\tau+\Theta)$.

Now transform the equation (2) with time-varying coefficients to an equation with constant coefficients. To this effect we use the Lyapunov transformation [7]

$$X(t,\tau)=F(\tau)\cdot Y(t,\tau). \tag{3}$$

Applying this transformation to (2) one obtains the following equation

$$\frac{\partial Y(t,\tau)}{\partial t}+\frac{\partial Y(t,\tau)}{\partial \tau}=KY(t,\tau)+N(\tau)\cdot B(t,\tau), \tag{4}$$

where $F(\tau)=F(\tau+\Theta)$ is a periodic matrix; $Y(t,\tau)$ is a new vector of state variables, $N(\tau)$ is an inverse matrix to $F(\tau)$, i.e. $N(\tau)=F^{-1}(\tau)$;

$$B(t,\tau)=\begin{pmatrix} \frac{s(\tau)}{L}\\ 0\end{pmatrix}e(t).$$

Matrices K and $F(\tau)$ are determined from the equation

$$\frac{dF(\tau)}{d\tau}=A(\tau)F(\tau)-F(\tau)K, \tag{5}$$

and conditions $F(\tau)=F(\tau+\Theta)$, $F(0)=I$ (I is the identity matrix).

The matrix $F(\tau)$ is a piecewise continuous matrix. Solving the equation (5) in the interval $0\le\tau\le t_1$ gives [8]

$$F(\tau)=e^{A_1\tau}e^{-K\tau},$$

and in the interval $t_1\le\tau\le\Theta$ gives

$$F(\tau)=e^{A_2(\tau-t_1)}e^{A_1t_1}e^{-K\tau}, \tag{6}$$

where A_1 is equal to the matrix $A(\tau)$ as $s(\tau)=1$, A_2 is equal to the matrix $A(\tau)$ as $s(\tau)=0$.

Substituting in (6) $\tau=\Theta$ and then $F(\Theta)=F(0)=I$ one finds the matrix

$$K=\frac{1}{\Theta}\ln\left[e^{A_2(\Theta-t_1)}e^{A_1t_1}\right].$$

The differential equation (4) is a partial differential equation and defines the vector $Y(t,\tau)$ in the domain of two variables.

III. Solving differential equation

Let us find a solution to (4). Using the two dimensional Laplace transform [9]

$$F(s,q)=\int_0^{\infty}\int_0^{\infty} f(t,\tau)e^{-st-q\tau}d\tau dt$$

to solve (4) one obtains the algebraic equation

$$sY(s,q)+qY(s,q)=K\cdot Y(s,q)+N(q)*B(s,q), \tag{7}$$

where s and q are complex variables; $Y(s,q)$, $N(q)$ and $B(s,q)$ are Laplace transforms of $Y(t,\tau)$, $N(\tau)$ and $B(t)$; * is the sign of the convolution in the frequency domain [10]

$$N(q)*B(s,q)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}N(p)B(s,q-p)dp.$$

In order to simplify a calculation of the convolution we take into account that the matrix $N(\tau)$ and the switching function $s(\tau)$ have the same the continuity interval and that in the first interval the function $s(\tau)$ is constant. So, one can transform the product as follows

$$N(\tau)B(t,\tau)=N(\tau)\begin{pmatrix}\dfrac{s(\tau)}{L}\\0\end{pmatrix}e(t)=P(\tau)e(t),$$

where $P(\tau)=N(\tau)\begin{pmatrix}\dfrac{s(\tau)}{L}\\0\end{pmatrix}$ is a vector that in the first column of the matrix $N(\tau)$ selects only the components correspondent to the first interval $0\le\tau\le t_1$.

Taking this into account one can write

$$sY(s,q)+qY(s,q)=KY(s,q)+P(q)E(s),$$

where $E(s)$ and $P(q)$ are Laplace transforms of the voltage $e(t)$ and the vector $P(\tau)$.

From (7) one obtains

$$Y(s,q)=[(s+q)-K]^{-1}P(q)E(s), \qquad (8)$$

where $\left[(s+q)I-K\right]^{-1}$ is the inverse matrix.

In order to find the vector $X(t,\tau)$ one must apply the Lyapunov transformation. Taking the Laplace transform of (3) one obtains

$$X(s,q)=F(q)*Y(s,q). \qquad (9)$$

Substituting (8) in (9), one obtains the solution

$$X(s,q)=F(q)*\{[(s+q)I-K]^{-1}P(q)\}E(s). \quad (10)$$

IV. FREQUENCY RESPONSES

The transfer function of the circuit as the quotient of $X(s,q)$ to $E(s)$ is

$$T(s,q)=\frac{X(s,q)}{E(s)}.$$

From (10) one obtains

$$T(s,q)=F(q)*\left\{\left[(s+q)I-K\right]^{-1}P(q)\right\}. \qquad (11)$$

Since $F(\tau)$ and $P(\tau)$ are periodical functions, then the Laplace transforms take forms

$$F(q)=\frac{\int_0^{\Theta}F(\tau)e^{-q\tau}d\tau}{1-e^{-q\Theta}}=\frac{F'(q)}{1-e^{-q\Theta}},$$

$$P(q)=\frac{\int_0^{t_1}P(\tau)e^{-q\tau}d\tau}{1-e^{-q\Theta}}=\frac{P'(q)}{1-e^{-q\Theta}}.$$

The convolution in (11) is calculated with respect to poles

$$q_i=j\frac{2\pi i}{\Theta}$$

of the transform $F(q)$ that are found by solving the equation

$$1-e^{-q_i\Theta}=0,\ i=0,\pm1,\pm2,...$$

Calculating residues at these poles yields

$$T(s,q)=$$

$$=\frac{\sum_{i=-\infty}^{\infty}F'(q_i)\left[(q-q_i+s)I-K\right]^{-1}P'(q-q_i)}{\Theta\left(1-e^{-q\Theta}\right)}. \qquad (12)$$

This transform is obtained by virtue of the periodicity of the vector $P(\tau)$.

Let us define frequency responses for the inverter. With that end we find a steady-state process for the voltage $e(t)=E_c\cos(\omega t)$. The Laplace transform of this voltage is

$$E(s)=\frac{s}{s^2+\omega^2}E_c,$$

where E_c is an amplitude of the voltage $e(t)$.

By multiplying $T(s,q)$ by $E(s)$ one finds the Laplace transform of the vector of state variables

$$X(s,q)=T(s,q)E(s).$$

Substituting $T(s,q)$ from (12) yields

$$X(s,q)=\frac{sE_c}{\Theta\left(1-e^{-q\Theta}\right)\left(s^2+\omega^2\right)}\cdot$$

$$\sum_{i=-\infty}^{\infty}F'(q_i)\left[(q-q_i+s)I-K\right]^{-1}P'(q-q_i).$$

The steady-state process is determined by finding the inverse Laplace transform

$$X_s(t,\tau)=\frac{1}{(2\pi j)^2}\int_{c-j\infty}^{c+j\infty}\int_{d-j\infty}^{d+j\infty}X(s,q)e^{st}e^{q\tau}dsdq,$$

with respect to poles corresponding to the control signal $q_n=j\dfrac{2\pi n}{\Theta}$ ($n=0,\pm1,\pm2,...$) and the power supply $s_m=jm\omega$ ($m=-1,1$). The steady-state behavior in the form of the double Fourier series [11] is founded by the calculation of residues at these poles

$$X_s(t,\tau)=\sum_{poles\, s_m}\sum_{poles\, q_n}\mathrm{Re}s\left[X(s,q)e^{st}e^{q\tau}\right]=$$

$$\sum_{\substack{m=-1\\m\ne0}}^{1}\sum_{n=-\infty}^{\infty}C_{m,n}e^{j(m\omega t+n\Omega\tau)},$$

where $\Omega=\dfrac{2\pi}{\Theta}$,

$$C_{m,n} = \frac{mE_c}{2\Theta^2} \sum_{i=-\infty}^{\infty} F'(q_i)\left[(q_n - q_i + j\omega m)I - K\right]^{-1}.$$
$$P'(q_n - q_i)\,.$$

Subscripts m and n take positive and negative values.

In order to introduce frequency responses let us rearrange coefficients $C_{m,n}$ as follows

$$C_{1,n}e^{j(\omega t + n\Omega\tau)} + C_{-1,-n}e^{-j(\omega t + n\Omega\tau)} +$$
$$C_{1,-n}e^{j(\omega t - n\Omega\tau)} + C_{-1,n}e^{j(-\omega t + n\Omega\tau)} =$$
$$A_{1,n}\sin\left(\omega t + n\Omega\tau + \phi_{1,n}\right) +$$
$$A_{1,-n}\sin\left(\omega t - n\Omega\tau + \phi_{1,-n}\right),$$

where $A_{1,n} = 2\cdot\left|C_{1,n}\right|$, $A_{1,-n} = 2\cdot\left|C_{1,-n}\right|$, $\phi_{1,n} = \arg C_{1,n}$, $\phi_{1,-n} = \arg C_{1,-n}$.

We define amplitude responses $A_{1,n}$, $A_{1,-n}$ and phase responses $\phi_{1,n}$, $\phi_{1,-n}$ as functions dependent on ω and Ω [12]. As one can see frequency responses are a set of both amplitudes and phases characteristics.

V. Results of calculation

Let us draw the amplitude and phase responses of the inverter circuit for parameter values: $E_c = 1V$, $R_L = 0.2\,\Omega$, $R = 800\Omega$, $L = 0.0015\,H$, $C = 6\cdot 10^{-6}\,F$, $\Theta = 7\cdot 10^{-4}\,s$, $t_1 = 2\cdot 10^{-4}\,s$. The frequency responses for $i = 8$ are shown in Fig. 3 - Fig. 6.

One can see that the amplitude responses show a resonance for different harmonics.

Let us compare obtained results with the averaged state-space method [13]. Taking the average of (1) gives

$$\frac{d\hat{X}(t)}{dt} = A(d)\hat{X}(t) + dB(t)\,,$$

where $\hat{X}(t) = \begin{pmatrix}\hat{i}(t)\\ \hat{u}(t)\end{pmatrix}$ is a vector of averaged state variables; $A(d) = \begin{pmatrix} -\frac{R_L}{L} & -\frac{1-d}{L} \\ \frac{1-d}{C} & -\frac{1}{RC}\end{pmatrix}$; $d = \frac{t_1}{\Theta}$.

Applying the Laplace transform to the obtained equation allows us to find the Laplace transform of the voltage

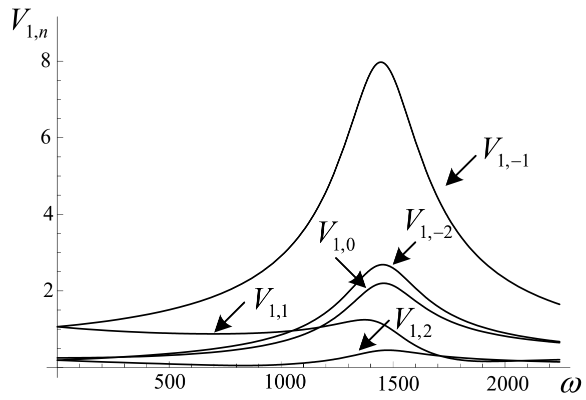

Fig. 3. Amplitude-voltage responses

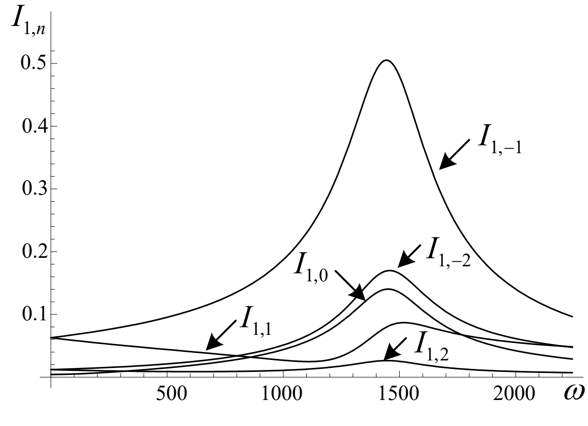

Fig. 4. Amplitude-current responses

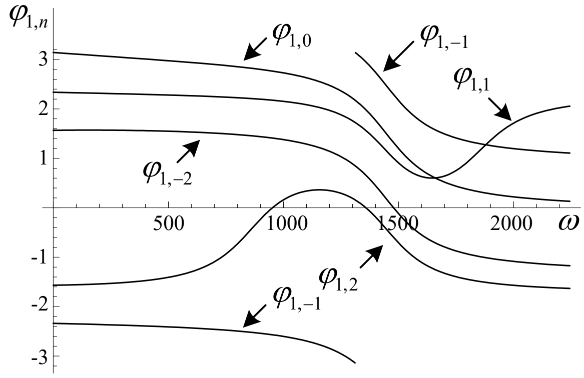

Fig. 5. Phase-voltage response

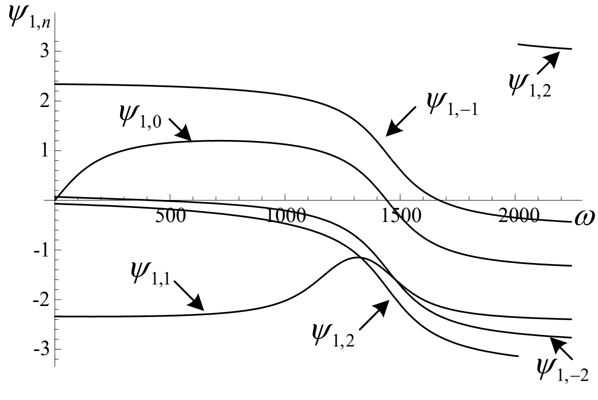

Fig. 6. Phase-current responses

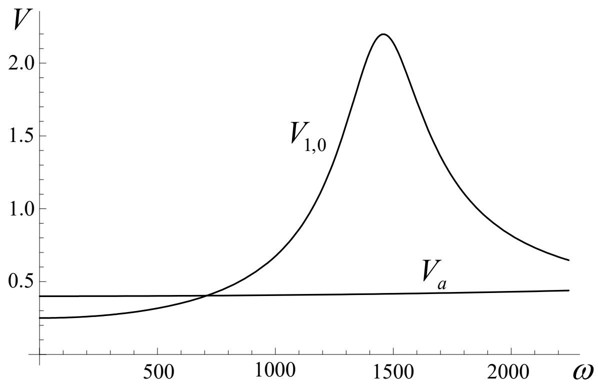

Fig. 7. The magnitude responses for first harmonic of the voltage

$$\hat{U}(s) = \frac{(1-d)dRE_c}{RCLs^2 + \left(RCR_L + L\right)s + R_L + R(1-d)^2}.$$

A magnitude response is determined by substituting $s \to j\omega$ in this formula. Amplitude responses for the voltage obtained by the proposed method $V_{1,0}$ and the averaged state-space method V_a are presented in the Fig. 7.

It should be emphasized that a difference between results arises in case of small time constants of the circuit.

In order to verify the obtained results equation (1) has been solved numerically for all intervals. The steady-state behavior has been calculated via the calculation of a transient process with zero initial conditions. For this purpose the NDSolve function of Mathematica has been used. The calculation has been carried out for the time interval from 0 to $600\,\Theta$ s. Also cosine and sine components for $\omega \pm n\Omega$ have been numerically calculated. Frequency responses have been calculated and compared with the proposed method for $T = 30\Theta$, $T = 10\Theta$, $T = 6\Theta$, $T = 4\Theta$. The accuracy of calculations is less than one percent.

CONCLUSION

In this paper the steady-state processes and frequency response have been determined for time varying circuits. In order to analyse processes in an inverter with periodic structure the differential equation has been expanded into the domain of two variables of time. For solving this equation the Lyapunov transformation and the two dimensional Laplace transform have been used. The solution for the steady-state behaviour has been represented by the double Fourier series. The coefficients of Fourier series have been used for definition of frequency responses. These characteristics are the response of the periodic time varying circuit to the sinusoidal signal source. The example of frequency responses for the buck-boost inverter has been presented. The obtained results have been compared with numerical calculations and the averaged state-space method. The obtained results show the presence of resonance that is not revealed by the use of the averaged state-space method.

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Анализ частотных характеристик инвертирующего преобразователя

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***Аннотация*—Статья посвящена анализу процессов в цепях инверторов. Процессы описываются дифференциальными уравнениями с периодическими коэффициентами. В виду того, что сигналы управления и напряжение, действующие в цепи инвертора независимы, их частоты часто являются несоизмеримыми. Для получения стационарных периодических решений, обыкновенные дифференциальные уравнения с одной независимой переменной времени расширены на уравнения в частных производных с двумя независимыми переменными времени. Полученные дифференциальные уравнения в частных производных с периодическими коэффициентами преобразуются в уравнения с постоянными коэффициентами с помощью преобразования Ляпунова. Поскольку эти уравнения определены в области двух переменных времени, их решения определяются с помощью двумерного преобразования Лапласа. После решения дифференциальных уравнений установившийся процесс представлен в виде двойного ряда Фурье. Частотные характеристики определяются в виде функций коэффициентов ряда Фурье, зависящих от частоты управляющего сигнала и напряжения питания. Представлены результаты расчетов частотных характеристик инвертирующего преобразователя.**

Библ. 11, рис. 7.

Ключевые слова* — *расширение дифференциальных уравнений, частотные характеристики, двумерное преобразование.

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Аналіз частотних характеристик інвертуючого перетворювача

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***Анотація*—Стаття присвячена аналізу процесів в ланцюгах інверторів. Процеси описуються диференційними рівняннями з періодичними коефіцієнтами. Через те, що сигнали управління і напруги, що діють в ланцюзі інвертора незалежні, їх частоти часто є непорівнянними. Для отримання стаціонарних періодичних рішень, звичайні диференційні рівняння з однією незалежною змінною часу розширені на рівняння в частинних похідних з двома незалежними змінними часу. Отримані диференційі рівняння в частинних похідних з періодичними коефіцієнтами перетворюються в рівняння з постійними коефіцієнтами за допомогою перетворення Ляпунова. Оскільки ці рівняння визначені в області двох змінних часу, їх рішення визначаються за допомогою двовимірного перетворення Лапласа. Після вирішення диференційних рівнянь процес, що встановився, представлений у вигляді подвійного ряду Фур'є. Частотні характеристики визначаються у вигляді функцій коефіцієнтів ряду Фур'є, що залежать від частоти керуючого сигналу і напруги живлення. Представлені результати розрахунків частотних характеристик інвертуючого перетворювача.**

Бібл. 11., рис. 7

Ключові слова* — *розширення диференційних рівнянь, частотні характеристики, двовимірне перетворення