Електронні системи та сигнали

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Steady-State Analysis of Inverter Working with Varied Load

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***Abstract*—The paper deals with an analysis of steady-state behaviours in inverter circuits. Transient and steady-state processes in inverter circuits are described by differential equations with periodical coefficients. The control of inverter switches and a load is realised by signals which frequencies are independent. These frequencies are incommensurable. In order to obtain steady-state periodic solutions ordinary differential equations are expanded into partial differential equations with two independent variables of time. The solution of partial differential equations is found by calculating a periodic boundary condition for an arbitrary period in the domain of two time variables. Obtained functions are used for forming a time delay equation. This equation is solved by the use of the Galerkin method with sinusoidal weight and basis functions. The result of a calculation is represented in the form of the Fourier series. The periodic boundary condition and steady-state process for a boost inverter with varied load are presented.**

Ref. 8, fig. 6.

Key words — expansion of differential equations; steady-state analysis; periodical boundary condition.

I. INTRODUCTION

The analysis of steady-state processes in periodic time-varying circuits could be realised either by a computation of a transient process or find an initial value of a steady-state process. Processes in such circuits can be analysed by different method [1, 2, 3, 4] in case these circuits are controlled by signals with the same or proportional frequencies.

For invertor circuits controlled by one frequency an initial value for the calculation of a steady-state process one can find by solving a differential equation on an arbitrary period and then equating initial and finite values. In case of incommensurable frequencies one cannot find an initial value as a number. The problem is based on that one cannot find a periodic steady-state behaviour.

Finding of the periodic steady-state behaviour can be realised by an expansion of ordinary differential equations with one time variable into partial differential equations with two time variables [5, 6, 7]. The expansion of nonlinear differential equations with different input sources and a process analysis in the circuit of a Buck converter with periodical supply source are considered in [6] and [7].

The aim of this article is to find a periodic boundary condition which can be used for calculation a steady-state behaviour. The method is based on the expansion of a differential equation and solving this equation for one period. As a result one obtains a time delay equation. This equation is solved by the use of the Galerkin method and a result is represented in the form of the Fourier series. An example of a boost inverter with a varied load is considered and results of computation are presented.

II. MATHEMATICAL MODEL

Let us consider the circuit of the boost inverter presented in Fig. 1. The switches S_1, S_2 and S_3 are ideal. Switches S_1 and S_2 switch alternately, if S_1 is closed, S_2 is open as shown in Fig. 2 (we consider that when S_1 is closed and S_2 is open).

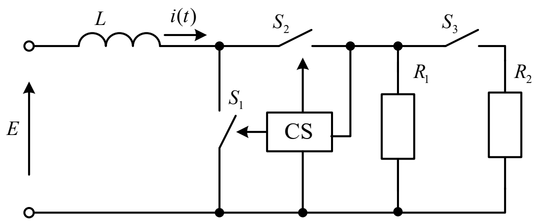

Fig. 1. Topology of the inverter

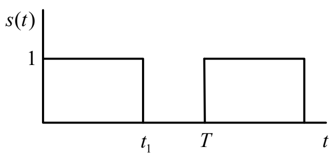

Fig. 2. Switching function of the inverter

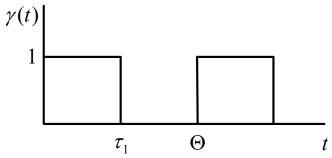

Fig. 3. Switching functions of the load

The switch S_3 is working as shown in Fig. 3 ($\gamma(t)=1$ corresponds to the close state).

The topology of the inverter is changed periodically. The differential equation describing processes in the circuit has the form

$$\frac{di(t)}{dt}=-\frac{R_L+s(t)\dfrac{R_1R_2}{\gamma(t)R_1+R_2}}{L}i(t)+\frac{E}{L}, \qquad (1)$$

where R_L is the resistance of an inductor, T and Θ are periods of switching functions.

In case of incommensurable periods of switching functions $s(t)$ and $\gamma(t)$ the steady-state behavior to (1) is not periodic.

We expand the differential equation (1) from one time variable t to the partial differential equation to two time variables t and τ as follows [5]

$$\frac{\partial i(t,\tau)}{\partial t}+\frac{\partial i(t,\tau)}{\partial \tau}=a(t,\tau)i(t,\tau)+b\,, \qquad (2)$$

where

$$a(t,\tau)=-\frac{R_L+s(t)\dfrac{R_1R_2}{\gamma(\tau)R_1+R_2}}{L},\ b=\frac{E}{L}.$$

It should be noted that the periodic steady-state process $i_s(t,\tau)=i_s(t+T,\tau+\Theta)$ exists for the expanded system. Since process is periodical we find a periodical boundary condition $i(t,\Theta)=i(t+T,\Theta)$. Using this boundary condition one can calculate the steady-state process.

III. SOLVING DIFFERENTIAL EQUATION

Let us find a solution to (2) for an arbitrary period in the domain of two variables. Assume that values of parameters t_1, T, τ_1 and Θ are such as shown in Fig. 4. In this figure we designate areas of simultaneous work of switching functions as I, II, III, IV, i.e. I corresponds to $s(t)=1$ and $\gamma(\tau)=1$, II - $s(t)=0$ and $\gamma(\tau)=1$, III - $s(t)=0$ and $\gamma(\tau)=0$, IV - $s(t)=1$ and $\gamma(\tau)=0$. Inclined lines *a*, *b*, *c d*, *e*, *f* and *g* delimit regions with the same solutions. We designate hereafter coefficients k_1,k_2,k_3,k_4 as follows

$$k_1=a(t,\tau)\big|_{s(t)=1,\gamma(\tau)=1},\ k_2=a(t,\tau)\big|_{s(t)=0,\gamma(\tau)=1},$$
$$k_3=a(t,\tau)\big|_{s(t)=0,\gamma(\tau)=0},\ k_4=a(t,\tau)\big|_{s(t)=1,\gamma(\tau)=0}.$$

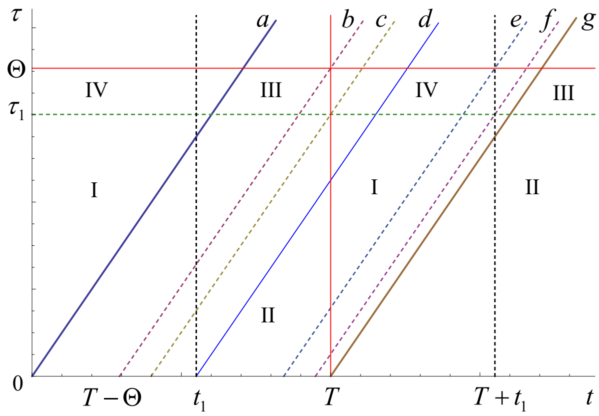

Fig. 4. Intervals of switching of the inverter and load

For the first interval (situated between lines *a* and *b*) the solution to (2) is found with respect to a boundary condition $i(t,0)$

$$i(t,\tau)=e^{k_1\tau}i(t-\tau,0)+\left(e^{k_1\tau}-1\right)b/k_1\,. \qquad (3)$$

This solution is defined for $t\ge\tau$.

In order to find the solution to (2) for the second interval we shift the coordinate origin to the point $(0,t_1)$. It should be noted that the solution is determined with respect to a boundary condition $i(0,\tau)$, so we obtain

$$i(t,\tau)=e^{k_2 t}i(0,\tau-t)+\left(e^{k_2 t}-1\right)b/k_2\,. \qquad (4)$$

In order to write this expression for the coordinate origin $(0,0)$ we substitute the end of boundaries $t\to t_1$ and $\tau\to\tau-t+t_1$ in (3), shift (4) as $t\to t-t_1$ and then substitute the first expression into the second so that we get

$$\begin{aligned} i(t,\tau)&=e^{k_2(t-t_1)}e^{k_1(\tau-t+t_1)}i(t-\tau,0)+\\ &+e^{k_2(t-t_1)}\left[e^{k_1(\tau-t+t_1)}-1\right]b/k_1+\\ &+\left[e^{k_2(t-t_1)}-1\right]b/k_2 \end{aligned} \qquad (5)$$

For the third interval the solution to (2) is found with respect to the boundary condition $i(t,0)$. For this purpose we shift the coordinate origin to the point (τ_1,t_1)

$$i(t,\tau)=e^{k_3\tau}i(t-\tau,0)+\left(e^{k_3\tau}-1\right)b/k_3\,. \qquad (6)$$

Now we shift this expression to the coordinate origin $(0,0)$. We substitute the end of boundaries $\tau\to\tau_1$ and $t\to t-\tau+\tau_1$ in (5), shift (6) as $\tau\to\tau-\tau_1$ and then substitute the first expression into the second, which yields

$$\begin{aligned} i(t,\tau)&=e^{k_3(\tau-\tau_1)}e^{k_2(t-\tau+\tau_1-t_1)}e^{k_1(\tau-t+t_1)}i(t-\tau,0)+\\ &+e^{k_3(\tau-\tau_1)}\left\{e^{k_2(t-\tau+\tau_1-t_1)}\left[e^{k_1(\tau-t+t_1)}-1\right]b/k_1\right. \end{aligned} \qquad (7)$$

In this expression $\Theta\le t\le T$.

From (7) we extract two parts: the function at $i(t-\tau,0)$ and the second term, i.e.

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$$f_1(t,\tau)=e^{k_3(\tau-\tau_1)}e^{k_2(t-\tau+\tau_1-t_1)}e^{k_1(\tau-t+t_1)}, \quad (8)$$

$$q_1(t,\tau)=e^{k_3(\tau-\tau_1)}\times$$
$$\times\left\{e^{k_2(t-\tau+\tau_1-t_1)}\left[e^{k_1(\tau-t+t_1)}-1\right]b/k_1+\right. \quad .(9)$$
$$\left.+\left[e^{k_2(t-\tau+\tau_1-t_1)}-1\right]b/k_2\right\}+\left[e^{k_3(\tau-\tau_1)}-1\right]b/k_3$$

In the same way we find solutions for intervals:

$T\le t\le\Theta+T-\tau_1$ (situated between lines *b* and *c*)

$$f_2(t,\tau)=e^{k_4(t-T)}e^{k_3(\tau-t+T-\tau_1)}e^{k_2(t-\tau+\tau_1-t_1)}\cdot e^{k_1(\tau-t+t_1)},$$

$$q_2(t,\tau)=e^{k_4(t-T)}\left\{e^{k_3(\tau-t+T-\tau_1)}\left\{e^{k_2(t-\tau+\tau_1-t_1)}\times\right.\right.$$
$$\times\left[e^{k_1(\tau-t+t_1)}-1\right]b/k_1+\left[e^{k_2(t-\tau+\tau_1-t_1)}-1\right]b/k_2\right\}+,$$
$$\left.+\left[e^{k_3(\tau-t+T-\tau_1)}-1\right]b/k_3\right\}+\left[e^{k_4(t-T)}-1\right]b/k_4$$

$\Theta+T-\tau_1\le t\le\Theta+t_1$ (situated between lines *c* and *d*)

$$f_3(t,\tau)=e^{k_4(\tau-\tau_1)}e^{k_1(t-\tau+\tau_1-T)}e^{k_2(T-t_1)}e^{k_1(\tau-t+t_1)},$$

$$q_3(t,\tau)=e^{k_4(\tau-\tau_1)}\left\{e^{k_1(t-\tau+\tau_1-T)}\left\{e^{k_2(T-t_1)}\times\right.\right.$$
$$\times\left[e^{k_1(\tau-t+t_1)}-1\right]b/k_1+\left[e^{k_2(T-t_1)}-1\right]b/k_2\right\}+ \quad ,$$
$$\left.+\left[e^{k_1(t-\tau+\tau_1-T)}-1\right]b/k_1\right\}+\left[e^{k_4(\tau-\tau_1)}-1\right]b/k_4$$

$\Theta+t_1\le t\le T+t_1$ (situated between lines *d* and *e*)

$$f_4(t,\tau)=e^{k_4(\tau-\tau_1)}e^{k_1(t-\tau+\tau_1-T)}e^{k_2(\tau-t+T)},$$

$$q_4(t,\tau)=e^{k_4(\tau-\tau_1)}\left\{e^{k_1(t-\tau+\tau_1-T)}\cdot\right.$$
$$\left[e^{k_2(\tau-t+T)}-1\right]b/k_2+\left[e^{k_1(t-\tau+\tau_1-T)}-1\right]b/k_1\right\}+$$
$$+\left[e^{k_4(\tau-\tau_1)}-1\right]b/k_4$$

$T+t_1\le t\le T+\Theta+t_1-\tau_1$ (situated between lines *e* and *f*)

$$f_5(t,\tau)=e^{k_3(t-T-t_1)}e^{k_4(\tau-t+T+t_1-\tau_1)}e^{k_1(t-\tau+\tau_1-T)}\cdot$$
$$e^{k_2(\tau-t+T)},$$

$$q_5(t,\tau)=e^{k_3(t-T-t_1)}\left\{e^{k_4(\tau-t+T+t_1-\tau_1)}\left\{e^{k_1(t-\tau+\tau_1-T)}\cdot\right.\right.$$
$$\left[e^{k_2(\tau-t+T)}-1\right]b/k_2+\left[e^{k_1(t-\tau+\tau_1-T)}-1\right]b/k_1\right\}+$$
$$\left.\left[e^{k_4(\tau-t+T+t_1-\tau_1)}-1\right]b/k_4\right\}+\left[e^{k_3(t-T-t_1)}-1\right]b/k_3,$$

$T+\Theta+t_1-\tau_1\le t\le T+\Theta$ (situated between lines *f* and *g*)

$$f_6(t,\tau)=e^{k_3(\tau-\tau_1)}e^{k_2(t-\tau-T+\tau_1-t_1)}e^{k_1t_1}e^{k_2(\tau-t+T)},$$

$$q_6(t,\tau)=e^{k_3(\tau-\tau_1)}\left\{e^{k_2(t-\tau-T+\tau_1-1)}\left\{e^{k_1t_1}\cdot\right.\right.$$
$$\left[e^{k_2(\tau-t+T)}-1\right]b/k_2+\left[e^{k_1t_1}-1\right]b/k_1\right\}+$$
$$\left.\left[e^{k_2(t-\tau-T+\tau_1-t_1)}-1\right]b/k_2\right\}+\left[e^{k_3(\tau-\tau_1)}-1\right]b/k_3.$$

Let us use obtained expression to define functions

$$f(t,\tau)=\begin{cases}f_1(t,\tau),\Theta\le t\le T,\\ f_2(t,\tau),T\le t\le\Theta+T-\tau_1,\\ f_3(t,\tau),\Theta+T-\tau_1\le t\le\Theta+t_1,\\ f_4(t,\tau),\Theta+t_1\le t\le T+t_1,\\ f_5(t,\tau),T+t_1\le t\le T+\Theta+t_1-\tau_1,\\ f_6(t,\tau),T+\Theta+t_1-\tau_1\le t\le T+\Theta.\end{cases},$$

$$q(t,\tau)=\begin{cases}q_1(t,\tau),\Theta\le t\le T,\\ q_2(t,\tau),T\le t\le\Theta+T-\tau_1,\\ q_3(t,\tau),\Theta+T-\tau_1\le t\le\Theta+t_1,\\ q_4(t,\tau),\Theta+t_1\le t\le T+t_1,\\ q_5(t,\tau),T+t_1\le t\le T+\Theta+t_1-\tau_1,\\ q_6(t,\tau),T+\Theta+t_1-\tau_1\le t\le T+\Theta.\end{cases}$$

Using these functions we can interlink boundary conditions by forming the following equation

$$i(t,\Theta)=f(t,\Theta)i(t-\Theta,0)+q(t,\Theta). \quad (10)$$

This equation is a time delay equation. The equation is periodical, i.e. $i(t,0)=i(t+T,0)$ but $T\ne\Theta$.

In order to find the solution we represent the boundary condition of the current $i(t,0)$ in the form of the Fourier series [8]

$$i(t,0)=\frac{a_0}{2}+\sum_{k=1}^{N}\left[a_k\cos(k\omega t)+b_k\sin(k\omega t)\right],(11)$$

where $\omega=\dfrac{2\pi}{T}$.

We also find the Fourier series for functions $f(t,\Theta)$ and $q(t,\Theta)$.

In order to find the periodical boundary condition we use the Galerkin method for finding the residuum of the equation (10)

$$R_i=i(t,\Theta)-f(t,\Theta)i(t-\Theta,0)-q(t,\Theta) \quad (12)$$

over the interval $0\le t\le T$.

We multiply (12) by the weight functions $\cos(k\omega t)$ and $\sin(k\omega t)$ and integrate obtained expressions as follows

$$\int_0^T R_i\cos(k\omega t)dt=0, \quad (13)$$

$$\int_0^T R_i\sin(k\omega t)dt=0. \quad (14)$$

By solving the set (13-14) one obtains coefficients for the expression (11). It should be noted that in this case $i(t,0)=i(t,\Theta)$ and $i(t,0)=i(t+T,0)$. The obtained solution (11) is used for the calculation of the steady-state current.

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IV. Results of calculation

Let us find the periodical boundary condition of the inverter circuit for parameter values: $E = 10V$, $R_L = 0.2\Omega$, $R_1 = 4.5\Omega$, $R_2 = 5.8\Omega$, $L = 0.1H$, $T = 0.1s$, $t_1 = 0.55T\,s$, $\Theta = T/\sqrt{2}\,s$, $\tau_1 = 0.85\Theta\,s$. In order to verify the obtained results equation (1) has also been solved numerically for all intervals. The steady-state process has been calculated via the calculation of a transient process with the zero initial condition. The calculation has been carried out for the time interval from 0 to 42 Θ s. The periodic boundary condition calculated for $N = 5$ and the steady state process for the current are shown in Fig. 5.

Vertical lines correspond to the moments of time $t = 29\Theta$, $t = 30\Theta$ and $t = 31\Theta$. One can see coincidences of the periodic boundary condition with the steady state process.

Let us find the steady-state current. Points in which the periodic boundary condition and steady state process coincide are initial values for the calculation of the steady-state process.

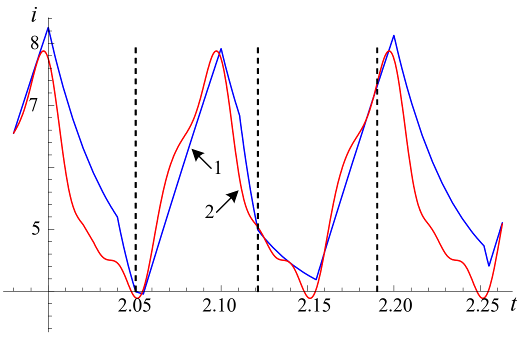

Fig. 5. Periodic boundary condition (2) and steady state current (1)

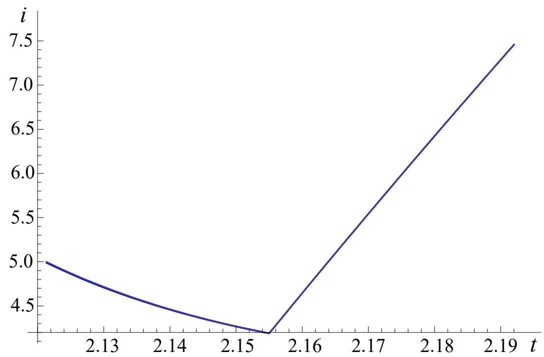

Fig. 6. Steady-state current for $30\Theta \le t \le 31\Theta$

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Let us chose the point for the boundary condition at $t = 30\Theta$. In this case the shift on the exes t is equal to the reminder of $\delta = 30\Theta \bmod T$ and the time shift is δT. Since δT is situated in the interval $0 \le t \le T - \Theta$ we use (4), and (7) for the calculation of the stead-state current. The steady-state current calculated by numerical and proposed method are presented in Fig. 6. One can see that these calculations are practically the same.

Conclusion

In this paper the periodic boundary condition has been determined for the time varying circuit. The ordinary differential equation has been expanded into the partial differential equation with two variables of time. Solving this equation allows to determine a time delay equation. The solution for the steady-state behaviour has been represented by the Fourier series. The example of calculation of the periodic boundary condition for the boost inverter is presented and compared with a numerical calculation of steady-state behaviour. Obtained results show good match for periodical values.

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Аналіз процесу перетворювача, що встановився зі змінним навантаженням

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***Анотація*—Стаття присвячена аналізу процесів, що встановилися, в ланцюгах інверторів. Процеси в ланцюгах інвертора описуються диференціальними рівняннями з періодичними коефіцієнтами. Управління ключами інвертора і навантаження здійснюється сигналами, частоти яких є незалежними. Відношення між частотами визначається ірраціональним числом. Для знаходження рішення, що встановилося, звичайні диференціальні рівняння розширюються на рівняння в приватних похідних з двома незалежними змінними часу. Рішення рівнянь в приватних похідних знаходиться шляхом обчислення періодичного граничного значення на довільному періоді в просторі двох змінних часу. Отримані функції використовуються для формування рівняння з часовою затримкою. Це рівняння вирішується за допомогою методу Галеркіна з синусоїдальними ваговими і базовими функціями. Результат розрахунку представлений у вигляді ряду Фур'є. Представлені результати розрахунків періодичного граничного значення і процесу, що встановився, для перетворювача, що підвищує, зі змінним навантаженням.**

Бібл. 8, рис. 6.

Ключові слова — розширення диференціальних рівнянь; аналіз процесу, що встановився; періодичне граничне значення

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Анализ установившегося процесса преобразователя с изменяемой нагрузкой

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***Аннотация*—Статья посвящена анализу установившихся процессов в цепях инверторов. Процессы в цепях инвертора описываются дифференциальными уравнениями с периодическими коэффициентами. Управление ключами инвертора и нагрузки осуществляется сигналами, частоты которых являются независимыми. Отношение между частотами определяется иррациональным числом. Для нахождения установившегося решения обыкновенные дифференциальные уравнения расширяются на уравнения в частных производных с двумя независимыми переменными времени. Решение уравнений в частных производных находится путем вычисления периодического граничного значения на произвольном периоде в пространстве двух переменных времени. Полученные функции используются для формирования уравнения с временной задержкой. Это уравнение решается с помощью метода Галеркина с синусоидальными весовыми и базовыми функциями. Результат расчета представлен в виде ряда Фурье. Представлены результаты расчетов периодического граничного значения и установившегося процесса для повышающего преобразователя с изменяемой нагрузкой.**

Библ. 8, рис. 6.

Ключевые слова – расширение дифференциальных уравнений; анализ установившегося процесса; периодическое граничное значение