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A Disturbed Grid Voltage Interharmonic Analysis with Fourier Series of Several Variables

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***Abstract*—Increasing the range of electronic devices, especially nonlinear impulse loads worsen the parameters of electricity quality. One of the most negative factors for the electricity quality are low-frequency interharmonics, which have a negative impact on electric motors, transformers and other AC equipment. The interharmonics generated by disturbances forming by grid loads which operates with frequency that is not multiple with the electric grid one. The interharmonics detection is a complex problem because of their permanent frequency fluctuation. The common numerical methods implementation for interharmonic analysis leads to large number of the mathematical operation, therefore analytical methods are more attractable. Usually, interharmonics forming is researched in wide range of disturbance parameters for further analysis and their suppression. Often, for math operation reducing, interharmonic analysis is performed by probabilistic approach in several points of parameters range with subsequent expanding its result on all parameter values that cause to inaccurate results. In the paper is proposed a fully analytical method based on a Fourier series of several variables, which allows: describe the voltage model immediately in the frequency domain; calculate the interharmonics of the voltage in all disturbance parameter range directly with a minimum number of mathematical operation due to representing the parameters in final analytical formula for interharmonics calculation; minimize error. In considered case the electric grid model consists of active-inductive internal resistance and a batch of nonlinear loads: AC system which operates on different frequency comparing with the grid frequency; pulse loads that switches with different frequency than grid frequency; parametric load with non-multiple period to the grid frequency. The properties of Fourier series of several variable are analyzed and advantages of describing signals with several components are shown. Principle of projection of multidimensional spectrum to one variable is demonstrated. The grid voltage disturbance for each load are analyzed and voltage model in frequency domain of several variable are obtained. The shape of each disturbance for batch of the input parameters are constructed and total grid voltage shape and its spectrum are built. The nature of interharmonic forming as combinational harmonics are shown. The mathematical operation elimination is analyzed for general case and shown the total advantage of the proposed method in comparing with numerical Fourier series of one variable.**

***Keywords* — grid voltage spectrum; interharmonic voltage disturbance; Fourier series of several variables.**

I. Introduction

Spreading of impulse nonlinear loads of AC power grid worsens the parameters of electricity quality. One of the negative factors caused by the impulse loads are interharmonics with frequency which is lower than the electric grid one [1]. The low-frequency interharmonics presence has negative impact on electric motors, transformers and other AC equipment [2].

Usually, interharmonics generated as combinational harmonics with frequencies f_{int}, as result of interaction of M processes, $M > 1$, with non-multiple frequencies f_1, f_2..f_M:

$$f_{int} = \sum_{k=1}^{M} m_k f_k , \qquad (1)$$

where m_k are integer numbers.

Measuring of the interharmonics because of permanent frequency changing is very complex problem and led to spectral leakage effect [3]. Therefore, the development of analysis method of disturbances impact on grid voltage that generate interharmonics is topical issue.

Implementation of common numerical methods for interharmonics detection caused to large amount of mathematical operations [4-6]. Often, for calculation reduction probabilistic methods are used [7,8], that allows calculate interharmonics and interpolate obtained results on all range of parameters values that together with significant reducing of math operation increase the calculation error. Whereas analytical methods allow reduce the amount operation without loss of accuracy.

In the proposed research a method of analytical representation of processes that generate interharmonics based on Fourier series of several variables [9] are proposed. The method allows to:

- obtain model of the signal directly in frequency domain that simplify interharmonic analysis;

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- calculate interharmonics for any parameters value with minimal math operation amount;
- minimize error due to implementing the fully analytical method without interpolation.

II. Power Grid Model

As usual, a source of grid current are consumers Consumer(1).. Consumer(k), that, because of the power grid non-zero inner resistance Z, caused grid voltage interharmonics in consumer connection point A. The power substitution scheme is shown in Fig. 1.

The voltage RMS value in point A, E_A is defined as difference of the grid voltage E on a generator output and the inner resistance Z, U_Z:

$$\dot{E}_A = \dot{E} - \dot{U}_Z = \dot{E} - \dot{Z}\left(\sum_k \dot{I}_{cust(k)}\right), \qquad (2)$$

where $I_{cust(k)}$ is current RMS value of the consumer k.

For voltage $e_A(t)$ calculation in time domain the Duhamel's integral is used [10]:

$$e_A(t) = e(t) - u_z(t) =$$
$$= e(t) - \left(h(t)\sum_k i_{cust(k)}(0) + \int_0^t h(t-\tau)\sum_k i_{cust(k)}{}'(\tau)d\tau \right), \qquad (3)$$

where $e(t)$ is voltage on the generator output, $u_Z(t)$ is voltage on inner resistance Z, $i_{cust(k)}(t)$ is instantaneous value of the consumer current k, $h(t)$ is voltage reaction on resistance Z on one-step current.

For household power grids inner resistance Z has active-inductor reaction, therefore voltage reaction $h(t)$ is described as follows:

$$h(t) = R + L\cdot\delta(t), \qquad (4)$$

where L, R are values of inductance and resistance of the inner resistance Z, δ is delta-function.

Let's consider typical current profiles that are generated by consumers with interharmonics.

III. Input Current Profiles

The consumers with interharmonics may be split into groups [1]:

1) AC systems that operate with frequency fint that are different from power grid frequency f1. Usually, such AC system connected to the grid via DC link. For instance, the another AC grid connected to the considered power grid via DC interconnector or asynchronous motor connected via electric drive [1]. As result, from the power grid consumed current iint, that defined as follows:

$$i_{int} = \sum_{k=1}^{l} I_{int(k)} \sin(2\pi k f_{int(1)} t + \varphi_k), \qquad (5)$$

where $I_{int(k)}$ is amplitude of current harmonic k, φ_k is phase of current harmonic k.

According to formula (4), voltage $u_Z(t)$ on inner resistance for defined in (5) current, is:

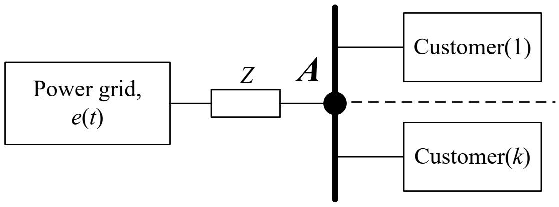

Fig. 1. The power grid substitution scheme

$$u_{Z1}(t) = R\sum_{k=1}^{l} I_{int(k)} \sin(2\pi k f_{int(1)} t + \varphi_k) -$$
$$-2\pi L k f_{int(1)} \sum_{k=1}^{l} I_{int(k)} \cos(2\pi k f_{int(1)} t + \varphi_k), \qquad (6)$$

2) Parametric loads: welding machines, laser printers with resistance R_{int} that has such time dependence:

$$R_{int} = R_0 \cdot f(t, f_{int}), \qquad (7)$$

where f is a modulation function with frequency f_{int}.

In the paper are analyzed two functional dependencies of parametric loads that cause:

- pulse current:

$$i(t) = \begin{cases} \dfrac{e(t)t}{R_{on}t_r}, 0 < t \le t_r; \\ \dfrac{e(t)}{R_{on}}, t_r < t \le t_r + \dfrac{\gamma_1}{f_{int(2)}}; \\ \dfrac{e(t)}{R_{on}} - \dfrac{\dfrac{e(t)}{R_{on}}\left(t - t_r - \dfrac{\gamma_1}{f_{int(2)}}\right)}{t_f}, \\ t_r + \dfrac{\gamma_1}{f_{int(2)}} < t \le t_r + t_f + \dfrac{\gamma_1}{f_{int(2)}}; \\ 0, t_r + t_f + \dfrac{\gamma_1}{f_{int(2)}} < t \le \dfrac{1}{f_{int(2)}}, \end{cases} \qquad (8)$$

where t_r, t_f are rise and fall edges of the current, R_{on} is a load pulse resistance, γ_1 is relative period duration of the load turn on;

- for smooth current:

$$i(t) = \begin{cases} e(t)\cdot G_0 t \dfrac{f_{int(3)}}{\gamma}, 0 < t \le \dfrac{\gamma_2}{f_{int(3)}}; \\ \dfrac{f_{int(3)} e(t) G_0 \left(\dfrac{1}{f_{int(3)}} - t\right)}{(1-\gamma_2)}, \dfrac{\gamma_2}{f_{int(3)}} < t \le \dfrac{1}{f_{int(3)}}, \end{cases} \qquad (9)$$

where expression $G_0 \cdot t$ is load conductance that smoothly changed with time, γ_2 is relative period duration when current is rise.

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Voltage u_Z on the grid inner resistance caused by current given in formulas (8) and (9) defined as follows:

$$u_{z3}(t)=\begin{cases} G_0 te(t)\dfrac{f_{int(3)}}{\gamma_2}R-\dfrac{G_0 f_{int(3)}L}{\gamma_2}, & 0<t\le\dfrac{\gamma_2}{f_{int(3)}};\\ \dfrac{f_{int(3)}G_0 e(t)R\left(\dfrac{1}{f_{int(3)}}-t\right)}{(1-\gamma_2)}+\dfrac{LG_0 f_{int(3)}}{(1-\gamma_2)}, & \dfrac{\gamma_2}{f_{int(3)}}<t\le\dfrac{1}{f_{int(3)}}; \end{cases} \quad (10)$$

$$u_{z2}(t)=\begin{cases} R\dfrac{e(t)t}{R_{on}t_r}-L\dfrac{I_{on}}{t_r}, & 0<t\le t_r;\\ R\dfrac{e(t)}{R_{on}}, & t_r<t\le t_r+\dfrac{\gamma_1}{f_{int(2)}};\\ R\dfrac{e(t)}{R_{on}}\left(1-\dfrac{\left(t-t_r-\dfrac{\gamma_1}{f_{int(2)}}\right)}{t_f}\right)+L\dfrac{I_{on}}{t_f}, & t_r+\dfrac{\gamma_1}{f_{int(2)}}<t\le t_r+t_f+\dfrac{\gamma_1}{f_{int(2)}};\\ 0, & t_r+t_f+\dfrac{\gamma_1}{f_{int(2)}}<t\le\dfrac{1}{f_{int(2)}}. \end{cases} \quad (11)$$

As result of the processes interaction (6), (10), (11) with frequencies $f_{int(k)}$ and power grid with frequency f_1 interharmonics are created. Let's consider advantages of Fourier series of several variables for interharmonics calculation.

IV. Voltage Representation Based Fourier Series of Several Variables

Fourier series of few variables describes signal in multidimensional space based on spectral coefficients $C_{(m1)(m2)..(mM)}$, where M is number of the variables $x_1=\omega_1 t$, $x_2=\omega_2 t,\ldots, x_M=\omega_M t$, where $\omega_1\ldots\omega_M$ are angular frequencies of the processes [11]:

$$C_{(m_1)(m_2)..(m_M)}=\frac{1}{2^{M-1}\pi^M}\times$$
$$\times\int_0^{2\pi}\ldots\int_0^{2\pi} y(x_1,x_2,..,x_M)e^{j\sum_{i=1}^{M}m_i x_i}\prod_{i=1}^{M}dx_i, \quad (12)$$

where $y(x_1, x_2,\ldots, x_M)$ is a function of M variables.

The Fourier series applying express's the spectral characteristics of the modulated signal in an analytical closed form for any multiplicity parameters $P_2,\ldots, P_M$, where $P_2=x_2/x_1,\ldots, P_M=x_M/x_1$. The signal harmonic with number k, C_k is calculated as follows [12]:

$$C_k=$$
$$=\sum_{m_2=-\infty}^{\infty}\sum_{m_M=-\infty}^{\infty}C_{(k-m_2P_2-m_3P_3\ldots-m_MP_M)(m_2)..(m_M)}. \quad (13)$$

As result of implementation of Fourier series of several variables the grid voltage is multiplied on disturbances (6), (10), (11), therefore for retrieving correct results the disturbances preliminarily have to be normalized on grid voltage $e(t)$:

$$u_Z^*(t)=1-\frac{u_Z(t)}{e(t)}. \quad (14)$$

After implementation of normalization (14) to (6), (10), (11), we obtain:

$$u_{Z1}^*(t)=1-\frac{R}{E_m}\sum_{k=1}^{l}I_{int(k)}\sin(2\pi kf_{int(1)}+\varphi_k)+\frac{2\pi Lkf_{int(1)}}{E_m}\sum_{k=1}^{l}I_{int(k)}\cos(2\pi kf_{int(1)}t+\varphi_k); \quad (15)$$

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$$u_{z2}^{*}(t)=\begin{cases}1-\frac{1}{E_m}\left(R\frac{t}{R_{on}t_r}-L\frac{1}{R_{on}t_r}\right), & 0<t\le t_r;\\ 1-\frac{R}{R_{on}E_m}, & t_r<t\le t_r+\frac{\gamma_1}{f_{int(2)}};\\ 1-\frac{1}{E_m}\left(\frac{R}{R_{on}}\left(1-\frac{\left(t-t_r-\frac{\gamma_1}{f_{int(2)}}\right)}{t_f}\right)+\frac{L}{R_{on}t_f}\right), & t_r+\frac{\gamma_1}{f_{int(2)}}<t\le t_r+t_f+\frac{\gamma_1}{f_{int(2)}};\\ 1, & t_r+t_f+\frac{\gamma_1}{f_{int(2)}}<t\le\frac{1}{f_{int(2)}};\end{cases} \quad (16)$$

$$u_{z3}^{*}(t)=\begin{cases}1-\frac{G_0}{E_m}\left(tR\frac{f_{int(3)}}{\gamma_2}-L\right), & 0<t\le\frac{\gamma_2}{f_{int(3)}},\\ 1-\frac{f_{int(3)}G_0}{E_m}\left(\frac{R\left(\frac{1}{f_{int(3)}}-t\right)}{(1-\gamma_2)}+\frac{L}{(1-\gamma_2)}\right), & \frac{\gamma_2}{f_{int(3)}}<t\le\frac{1}{f_{int(3)}}.\end{cases} \quad (17)$$

The voltage expression with interharmonic disturbances (15)-(17) based on Fourier series in four variables space: x_1 is argument of the grid voltage $e(x_1)=E_m\sin(x_1)$, x_2-x_4 are disturbances arguments $u_{z1}^{*}(x_2)$, $u_{z2}^{*}(x_3)$, $u_{z3}^{*}(x_4)$ is:

$$C_{(m_1)(m_2)(m_3)(m_4)}=\frac{1}{2^3\pi^4}\int_0^{2\pi}E_m\sin(Vx_1)e^{jm_1x_1}dx_1\int_0^{2\pi}u_{z1}^{*}(x_2)e^{jm_2x_2}dx_2\times$$
$$\times\int_0^{2\pi}u_{z2}^{*}(x_3)e^{jm_3x_3}dx_3\int_0^{2\pi}u_{z3}^{*}(x_4)e^{jm_4x_4}dx_4, \quad (18)$$

where V is lowest common multiple of $P_2 = x_2/x_1$, .., $P_M = x_M/x_1$:

$$V=Lcm(P_2;...;P_M). \quad (19)$$

Let's integrate separate parts in formula (18):

$$\text{int}_1=\int_0^{2\pi}E_m\sin(Vx_1)e^{jm_1x_1}dx_1=\begin{cases}j\pi E_m, & m_1=V;\\ -j\pi E_m, & m_1=-V;\\ 0, & m_1\ne -V, m_1\ne V;\end{cases} \quad (20)$$

$$\text{int}_2=\int_0^{2\pi}u_{z1}^{*}(x_2)e^{jm_2x_2}dx_2=\begin{cases}-\frac{\pi I_{int(k)}e^{-j\varphi_k}}{E_m}\left(jR+2\pi Lkf_{int(1)}\right), & m_2=k;\\ -\frac{\pi I_{int(2k-1)}e^{-j\varphi_k}}{E_m}\left(-jR+2\pi Lkf_{int(1)}\right), & m_2=-k;\\ 2\pi, & m_2=0;\\ 0, & m_2\ne 0, m_2\ne k, m_2\ne -k;\end{cases} \quad (21)$$

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$$\mathrm{int}_3 = \int_0^{2\pi} u_{z2}^{*}(x_3) e^{jm_3 x_3} dx_3 =$$

$$= \begin{cases} -\dfrac{1}{E_m R_{on}} \left(\begin{aligned} & R\frac{e^{j\varphi_r m_3}(1-j\varphi_r m_3)-1}{\varphi_r m_3^2} - \frac{jL}{\varphi_r m_3}\left(1-e^{jm_3\varphi_r}\right) + \\ & +R\frac{j\left(e^{jm_3\varphi_r} - e^{jm_3(\varphi_r+2\pi\gamma_1)}\right)}{m_3} + \\ & +R\left(\begin{aligned} & j\left(\frac{1}{m_3}+\frac{\varphi_r+2\pi\gamma_1}{\varphi_f m_3}\right) e^{jm_3(\varphi_r+2\pi\gamma_1)}\left(1-e^{jm_3\varphi_f}\right) - \\ & -\left(\begin{aligned} & e^{j(\varphi_r+\varphi_f+2\pi\gamma_1)m_3}\frac{\left(1-j\left(\varphi_r+\varphi_f+2\pi\gamma_1\right)m_3\right)}{\varphi_f m_3^2} - \\ & -e^{j(\varphi_r+2\pi\gamma_1)m_3}\frac{\left(1-j\left(\varphi_r+2\pi\gamma_1\right)m_3\right)}{\varphi_f m_3^2} \end{aligned} \right) \end{aligned} \right) + \\ & +\frac{jLe^{jm_3(\varphi_r+2\pi\gamma_1)}}{\varphi_f m_3}\left(1-e^{jm_3\varphi_f}\right) \end{aligned} \right), & m_3 \neq 0; \\ 2\pi - \dfrac{R}{E_m R_{on}}\left(\dfrac{\varphi_r}{2}+2\pi\gamma_1+\dfrac{\varphi_f}{2}\right), & m_3 = 0; \end{cases} \quad (22)$$

$$\mathrm{int}_4 = \int_0^{2\pi} u_{z3}^{*}(x_4) e^{jm_4 x_4} dx_4 = \begin{cases} -\dfrac{G_0}{E_m} \left(\begin{aligned} & R\frac{e^{jm_4 2\pi\gamma_2}(1-jm_4 2\pi\gamma_2)-1}{2\pi\gamma_2 m_4^2} - \frac{jL}{2\pi\gamma_2 m_4} \times \\ & \times\left(1-e^{jm_4 2\pi\gamma_2}\right) + \frac{j}{m_4}\left(\frac{R+L/2\pi}{(1-\gamma)}\right)\left(e^{jm_4 2\pi\gamma_2}-1\right) - \\ & -\frac{R\left((1-j2\pi m_4)-e^{jm_4 2\pi\gamma_2}(1-jm_4 2\pi\gamma_2)\right)}{2\pi(1-\gamma_2)m_4^2} \end{aligned} \right), m_4 \neq 0; \\ 2\pi - \dfrac{2G_0 R}{E_m}, m_4 = 0. \end{cases} \quad (23)$$

Based on obtained integrals int_1-int_4 in formulas (20)-(23) and values of V, P_2-P_4 spectrum of grid voltage $e_A(t)$ in connection point A are calculated.

V. Calculation of Grid Voltage Spectrum

A harmonic k value C_k is a sum of two combinational components:

$$k^{+} = \sum_{l=2}^{M} m_l P_l + V; \quad (24)$$

$$k^{-} = \sum_{l=2}^{M} m_l P_l - V, \quad (25)$$

each of them calculated by formula:

$$C_{\sum\limits_{l=2}^{M} m_l P_l \pm V} = \frac{1}{2^{M-1}\pi^{M}} \sum_{m_2=-\infty}^{\infty} \mathrm{int}_1 \ldots \sum_{m_M=-\infty}^{\infty} \mathrm{int}_M . \quad (26)$$

Based on formula (26) impact of each disturbance on the voltage in connection point of consumers $e_A(t)$ is obtained. Impact of disturbances is obtained for such value of the power grid parameters $e(t)$= =311sin(2π·50t)V, L = 100 μHn, R = 1 Ohm.

A disturbance caused by AC system with another frequency connected via DC link with parameters $f_{int(1)}$= =100/3 Hz, V = 3, φ = 0 with amplitude I_{int} = 30 A is shown in Fig. 2.

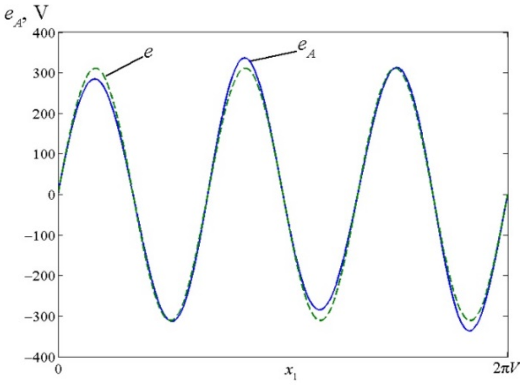

Fig. 2. A disturbance caused by AC system with another frequency

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A disturbance caused by an impulse load with resistance R_{on} = 0.01 Ohm connected with frequency $f_{int(2)}$ = 350/3 Hz, V = 3 to the power grid and parameters $\varphi_r = \varphi_f = 0.1\pi$, $\gamma_1 = 0.1$ is shown in Fig. 3.

A disturbance caused a load with smoothly changing in time conductance that changed periodically with frequency $f_{int(3)}$ = 650/3 Hz, G_0 = 100 Sm/s, V = 3 and $\gamma_2 = 0.7$ is shown in Fig. 4.

A total impact of the considered disturbances of grid voltage e_A and its spectrum calculated with formula (26) are shown in Fig. 5 a) and 5 b) respectively.

As shown in Fig. 5 a) the total voltage e_A is product of voltage e and disturbances (15)-(17). In Fig. 5 b) the harmonic amplitude that corresponds grid frequency has number $k = V = 3$. Harmonics with number k not multiple V, $k \neq V$, are interharmonics. With changing disturbances parameters spectrum calculation is more simple based on proposed method than with Fourier series of one variable. Let's define numerical indicator of calculation efficiency

VI. Efficiency comparing with Fourier series of one variable

Implementation of Fourier series of several variables for interharmonics calculation has such advantages comparing with Fourier series of one variable:

- direct spectrum calculation without signal representation in time domain;
- simple procedure of the spectrum recalculation when frequency of the disturbance is changed;
- splitting of each disturbance impact on voltage spectrum with possibility of its further analysis in analytical form.

One of the main advantage of the proposed method is significant reducing of math operation for spectrum calculation that allows use it in control system in real time scale.

Let's analyze the math operation reducing for the considered disturbances. For their representation g = 16 parameters: L, R, $f_{int(1)}$, φ, I_{int}, R_{on}, $f_{int(2)}$, φ_r, φ_f, γ_1, G_0, $f_{int(3)}$, γ_2, P_2, P_3, P_4 are used. Assume, that k = 1000 harmonics are calculated for l = 100 values of each parameter.

When Fourier series of one variable is used for spectrum calculation for overlaying disturbances the amount and positions of integration intervals always changed. Therefore, grid voltage has to be reconstructed in time domain again and its spectrum may be calculated only numerically. Such voltage representation requires define grid voltage at least in $2k$ points for every parameter set that needs

$$N_{(1)1} = 2kl^g F_1, \qquad (26)$$

operations, where F_1 is operation amount for voltage calculation in one point.

For spectrum calculation based on fast Fourier transform additionally operations executed

$$N_{(1)2} = k\log(k)l^g, \qquad (27)$$

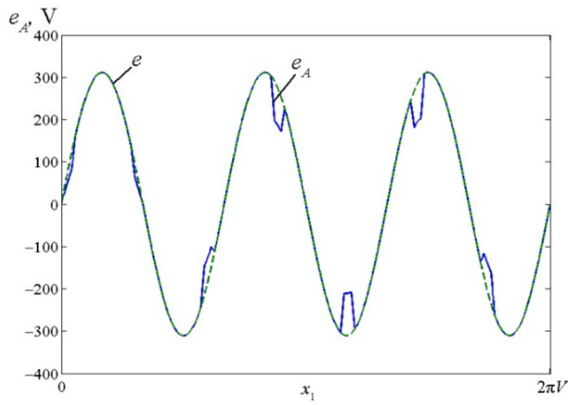


Fig. 3. A disturbance caused impulse load with resistance R_{on}

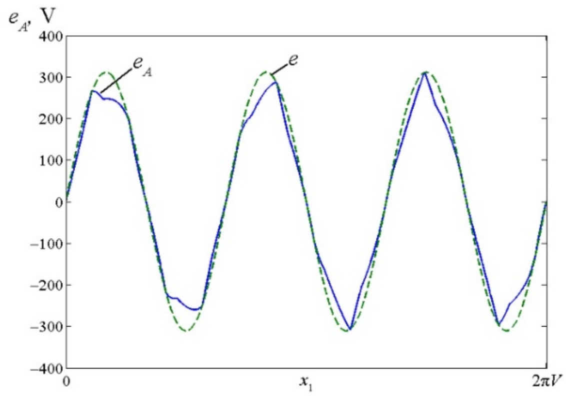
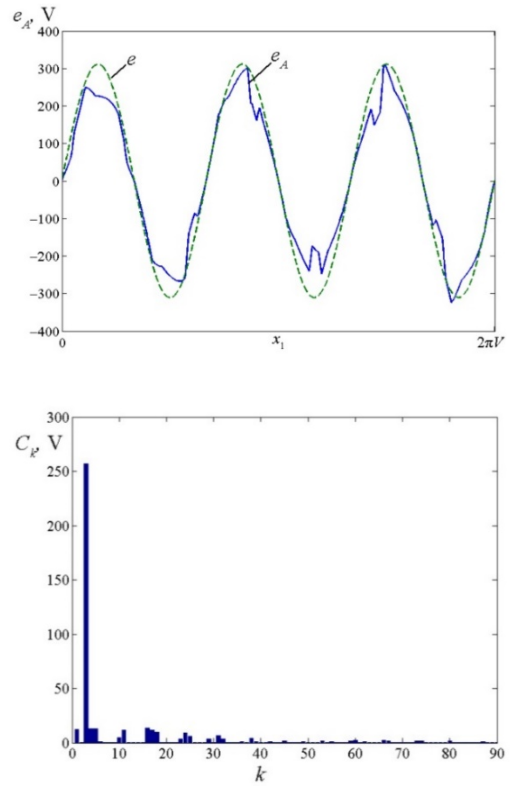


Fig. 4. A disturbance caused a load with smoothly changing in time conductance

a)



б)

Fig. 5. Total voltage that taking to account all disturbances: a) in time domain; b) in frequency domain

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When Fourier series of several variables is used, maximum value of coefficients m_{2_max}, m_{3_max}, m_{4_max} calculated by formula:

$$m_{\max} = \sqrt[M-1]{k} = 10. \quad (28)$$

As result, the voltage spectrum calculated with $N_{(2)1}$ operations:

$$N_{(2)1} = Mm_{\max} F_2, \quad (29)$$

where F_2 is average amount of math operations for component $\mathrm{int}_1..\mathrm{int}_M$ calculation, formula (26).After comparing an operation amount in (6), (10), (11) and (20)-(23) we can conclude that $F_1 \approx F_2$. Because each component $\mathrm{int}_1..\mathrm{int}_M$ depends on less parameters number, in our case: int_1 doesn't depend on disturbance parameters; int_2 depends on four parameters; int_3 depends on 6 parameters; int_4 depends on four parameters. We can assume, that each component $\mathrm{int}_1..\mathrm{int}_M$ approximately depends on g_f parameters, $g_f \approx g/M$. Therefore, component $\mathrm{int}_1..\mathrm{int}_M$ recalculated in fewer cases comparing with Fourier series of one variable. As result, a total amount of operation $N_{(2)t}$ for Fourier series of several variable implementation is equal to:

$$N_{(2)t} = N_{(2)1} M l^{g_f}. \quad (30)$$

The reducing of operation amount O_M for Fourier series of several variables is defined as interrelation of sum of expression (26) i (27) to expression (30):

$$O_M = \frac{l^{g(1-1/M)} k^{1-1/(M-1)}}{M^2} \left(2 + \log(k)/F_2\right) >> 1, \quad (31)$$

If assume that $F_2 = 30$, $O_M = 1.4 \cdot 10^{25}$, that prove very high efficiency of the proposed method. With increasing of the disturbance number and subsequently increasing of parameter M, parameter O_M will be increased exponentially. Therefore, implementation of the Fourier series of several variables for signal calculation, which formed as a result of the interaction of several processes with interharmonic generation is very useful and time-efficient.

Conclusions

In the paper the method of Fourier series of several variable implementation for the grid voltage spectrum calculation with disturbances that cause interharmonics is proposed. It is proved, that proposed method has such advantages:

- direct spectrum calculation without signal representation in time domain;
- simple procedure of the spectrum recalculation when frequency of the disturbance is changed;
- splitting of each disturbance impact on voltage spectrum with possibility of them further analysis in analytical form,
- and significantly reducing amount of math operation, that for considered in the paper case is $1.4 \cdot 10^{25}$ times comparing with Fourier series of one variable.

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References

[1] Testa, A., Akram, M. F., Burch, R., Carpinelli, G., Chang, G., Dinavahi, V., … Xu, W, "Interharmonics: Theory and Modeling," *IEEE Transactions on Power Delivery*, 2007, 22(4), pp. 2335–2348. - **DOI**: 10.1109/tpwrd.2007.905505.

[2] Yacamini, R, "Power system harmonics. Part 4: Interharmonics," *Power Engineering Journal*, 1996, 10(4), pp. 185–193. **DOI**: 10.1049/pe:19960411.

[3] Harris, Fredric j, "On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform," *Proceedings of the IEEE*, 1978, 66(1), pp. 51–83. **DOI**: 10.1109/PROC.1978.10837.

[4] N.E. Huang; N.O. Attoh-Okine, "The Hilbert-Huang transform in engineering," *Taylor & Francis*, 2005, 328 p. **DOI**: 10.1201/9781420027532.

[5] Tripathy, P.; Srivastava, S. C. & Singh, S. N., "A Modified TLS-ESPRIT-Based Method for Low-Frequency Mode Identification in Power Systems Utilizing Synchrophasor Measurements," *IEEE Transactions on Power Systems*, 2011, 26(2), pp. 719–727. **DOI**: 10.1109/tpwrs.2010.2055901.

[6] G.E.P. Box, G. M. Jenkins, G. C. Reinsel, "Time series analysis: Forecasting and control," 3rd ed., *Prentice-Hall International*, 1994, 712 p.

[7] Carbone, R., Castaldo, D., Langella, R., & Testa, A., "Probabilistic modeling of industrial systems for voltage distortion analyses," *Ninth International Conference on Harmonics and Quality of Power. Proceedings*, 2000. **DOI**: 10.1109/ichqp.2000.897748.

[8] Carbone, R., Castaldo, D., De Rosa, F., Langella, R., & Testa, A., "Monte Carlo simulation of AC/DC/AC power converter distortion," 2001 IEEE Porto Power Tech Proceedings, 2001. **DOI**: 10.1109/ptc.2001.964768.

[9] Verbytskyi, I.V.; Zhuikov, V.J., "Asynchronous motor drive interharmonics calculation based on generalized Fourier series of several variables," *Technical Electrodynamics*, 2020, № 2, pp. 36-42. **DOI**: 10.15407/techned2020.02.036.

[10] Anil K. Chopra, "Dynamics of Structures - Theory and applications to Earthquake Engineering," *Pearson Education Asia Limited and Tsinghua University Press*, Beijing, 2001.

[11] Ievgen Verbytskyi, "A double fourier series implementation for modulated signal with the arbitrary modulation," *2017 IEEE First Ukraine Conference on Electrical and Computer Engineering (UKRCON)*, 2017, pp. 518-521. **DOI**: 10.1109/UKRCON.2017.8100295.

[12] Verbytskyi, I. V., "Input Current Quality Parameters Analysis of Modular AC-DC SEPIC Charger Based on Double Fourier Series," Microsystems, Electronics and Acoustics, 2019, 24(6), p. 29-37. **DOI**: 10.20535/2523-4455.2019.24.6.197361.

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Інтергармонічний аналіз напруги мережі зі збуреннями на основі ряду Фур’є декількох змінних

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***Анотація*—Збільшення типів електронних пристроїв, особливо нелінійних імпульсних навантажень, погіршує параметри якості електроенергії. Одним з найбільш негативних факторів для якості електроенергії є низькочастотні інтергармоніки, які негативно впливають на електродвигуни, трансформатори та інше обладнання змінного струму. Інтергармоніки, викликані збуреннями, що утворюються за рахунок навантажень, які працюють з частотою, не кратною електричній мережі. Виявлення інтергармонік є складною проблемою через постійне коливання їх частоти. Впровадження загальних чисельних методів для інтергармонічного аналізу призводить до великої кількості математичних операцій, тому аналітичні методи є більш привабливими при вирішенні цієї задачі. Зазвичай, інтергармоніки досліджуються в широкому діапазоні параметрів збурень з метою подальшого аналізу та їх придушення. Для зменшення кількості математичних операцій під час інтергармонічного аналізу використовується імовірнісний підхід, що застосовується у кількох точках діапазону параметрів з подальшим розширенням його результату на весь діапазон параметрів, що призводять до неточних результатів. У статті пропонується повністю аналітичний метод, заснований на ряді Фур'є декількох змінних, що дозволяє: безпосередньо описати модель напруги в частотній області; обчислювати інтергармоніки напруги в усьому діапазоні параметрів збурень з мінімальною кількістю математичних операцій за рахунок введення цих параметрів у кінцевій аналітичній формулі для обчислення інтергармонік; мінімізувати помилку обчислень. Модель електричної мережі складається з активно-індуктивного внутрішнього опору та набору нелінійних навантажень: системи змінного струму, яка працює на іншій частоті порівняно з частотою мережі; імпульсне навантаження, що перемикається з іншою частотою, ніж частота мережі; параметричне навантаження з не кратним періодом зміни до частоти мережі. Проаналізовано властивості ряду Фур'є з декількох змінних та показано переваги опису сигналів з декількома компонентами. Показано принцип проекції багатовимірного спектру на одну змінну. Проаналізовано спотворення напруги мережі для кожного навантаження та отримано модель напруги в частотній області декількох змінних. Побудовано форму кожного збурення для діапазону вхідних параметрів та побудовано загальну форму напруги мережі та її спектр. Показано характер формування інтергармонік як комбінаційних гармонік. Аналізується зменшення кількості математичних операцій для загального випадку і показано загальну перевагу запропонованого способу у порівнянні з числовими рядом Фур'є однієї змінної.**

Ключові слова — спектр напруги мережі; інтергармонічні збурення напруги; ряд Фур’є декількох змінних