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Strategies of Energy-Efficient Active Filtering in the Two Wattmeter Reference Frame

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***Abstract*—Such basic concepts of power theory as apparent power, active current and minimum power losses in the transmission line are defined for a three-phase three-wire system in the shortened two wattmeter reference frame. On the basis of these concepts, two energy-efficient active filtration strategies were developed and verified by virtual experiments. The first of them minimizes the power losses of a three-phase three-wire transmission line with different values of its resistances and ensures a unit value of the power factor. The second strategy follows the recommendation of IEEE Std. 1459-2010 ensures the minimum power loss of the transmission line with symmetrical sinusoidal currents of a three-phase source.**

Keywords — three-phase three-wire power supply system; apparent power; active current; energy-efficient strategies of active filtering; two wattmeter reference frame.

1. Introduction

Semiconductor shunt active filters (SAF) are an effective means to improve the power quality on transformers substation's 6/0.4 kV busbars by compensating of inactive power components [1]. The SAF control strategies are based on the instantaneous and integral power theories, an overview of which is presented in [2, 3]. Power theories which use instantaneous current and voltage values originate from the pq instantaneous power theory proposed by H. Akagi at al. [4] for a three-phase three-wire system. In this theory, the analysis of energy processes is carried out in the *αβ* reference frame, which coordinates are converted from the three-coordinate current and voltage vectors by using matrix transformations. Further improvements and modifications of instantaneous power theories were carried out in the αβ0 [5], pqr [6] reference frames. Later, cross-vector [7] and vector [8] theories that do not require coordinate axes appear. Their comparative analysis was carried out in [8]. The power theory using integral quantities was initiated in the works of S. Fryze [9, 10], and acquired more advanced forms within the Fryze–Buchholz–Depenbrock Power Theory (FBD Theory [11]), Currents' Physical Components Power Theory (CPC Theory by L. Czarnecki [12]), Conservative Power Theory (CPT by Tenti and Mattavelli [13]), a comparative analysis of which is carried out in [14]. In a three-phase three-wire system, all the mentioned power theories operate with three-coordinate vectors of line currents and phase voltages calculated from the point of artificial grounding [11]. The three coordinates of each specified vectors are linearly dependent, which creates an unjustified complexity of the SAF control system due to an excessive number of sensors and regulators and the need to organize an artificial grounding point. In [15] it is proposed to control the SAF of a three-phase three-wire power system with the direct use of two-coordinate voltage and current vectors, the electrical variables of which have long been used to measure active power by the two wattmeters method [16]. In [17, 18], several strategies for SAF control were developed in the two wattmeter reference frame (TWRF), which have a number of advantages compared to the strategies of the instantaneous power theory. However, the maximum energy-saving effect of SAF application can be achieved only with the use of active filtering strategies using integral power theory [19].

The goal of the article is to develop energy-efficient strategies for shunt active filtering in TWRF, which ensure minimal power losses in the transmission line of a three-phase three-wire system, taking into account the requirements of modern standards for the power quality at the points of common coupling (PCC).

2. Active Filtration Strategy That Minimizes Power Losses in the Transmission Line

Fig. 1 shows a three-phase three-wire power system with resistive transmission line model and SAF enabled at PCC *A*, *B*, *C*. As shown in [17, 18], the energy processes of such a system can be comprehensively represented in TWRF with two-coordinate vectors of instantaneous line voltages calculated relative to a common point $\mathbf{u}(t) = \|u_{AC} \quad u_{BC}\|^{\wedge}$, where ^ – is the sign of the transposition, and the corresponding instantaneous currents $\mathbf{i}(t) = \|i_A \quad i_B\|^{\wedge}$. Taking into account the current equation $i_A + i_B + i_C = 0$ the connection of TWRF variables in the absence of SAF is established in the following vector-matrix form

$$\mathbf{u}(t) = \mathbf{e}(t) - \mathbf{Ri}(t), \tag{1}$$

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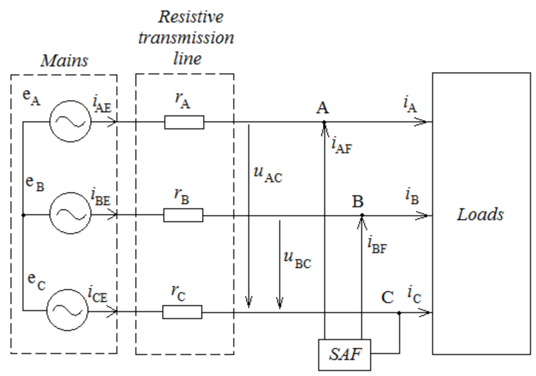

Fig. 1. Three-phase three-wire power supply system with a resistive model of power transmission line

where $\mathbf{R}=\begin{Vmatrix} r_A+r_C & r_C \\ r_C & r_B+r_C \end{Vmatrix}=\begin{Vmatrix} r_A & 0 \\ 0 & r_B \end{Vmatrix}+r_C\begin{Vmatrix}1\\1\end{Vmatrix}\begin{Vmatrix}1\\1\end{Vmatrix}^{\wedge}$ – is the matrix of transmission line resistances, symmetrical with respect to the main diagonal, $\mathbf{e}=\begin{Vmatrix} e_A-e_C & e_B-e_C \end{Vmatrix}^{\wedge}$ – is the vector of loop EMFs.

The load active power is determined by the two wattmeter method [16] using the notation for the scalar product of arbitrary T-periodic same-dimensional vectors $\mathbf{x}\circ\mathbf{y}=T^{-1}\int_T \mathbf{x}^{\wedge}(t)\mathbf{y}(t)dt$ as follows

$$P=T^{-1}\int_T[u_{AC}(t)i_A(t)+u_{BC}(t)i_B(t)]dt= \\ =T^{-1}\int_T \mathbf{u}^{\wedge}(t)\mathbf{i}(t)dt=\mathbf{u}\circ\mathbf{i}. \tag{2}$$

Power losses at the transmission line resistances in the absence of an active filter are

$$P_{LS}=T^{-1}\int_T\{i_A^2(t)r_A+i_B^2(t)r_B+i_C^2r_C\}dt= \\ =T^{-1}\int_T\begin{Vmatrix} i_A & i_B \end{Vmatrix}\left(\begin{Vmatrix} r_A & 0 \\ 0 & r_B \end{Vmatrix}+r_C\begin{Vmatrix}1\\1\end{Vmatrix}\begin{Vmatrix}1\\1\end{Vmatrix}^{\wedge}\right)\begin{Vmatrix} i_A \\ i_B \end{Vmatrix}dt= \\ =T^{-1}\int_T \mathbf{i}^{\wedge}(t)\mathbf{R}\mathbf{i}(t)dt=\mathbf{i}\circ\mathbf{R}\mathbf{i}. \tag{3}$$

Exactly power losses must figure in the apparent power definition formula to which energy-efficient active filtration strategies are linked. Almost identical formulas for the apparent power of multiphase systems with different values of transmission line resistances are known [20], [21]. Its are based on the FBD method with the corresponding disadvantages consist in using linearly dependent current and voltage vectors and the need to organize an artificial grounding point for calculating phase voltages. These shortcomings were overcame in [19], where the formula for apparent power of three-phase four-wire system was expressed through a three-coordinate vector of phase voltages calculated relative to the neutral wire and a three-coordinate vector of line currents. For this purpose Schwartz's inequality was used in the form

$$(\mathbf{R}^{-1/2}\mathbf{u}\circ\mathbf{R}^{1/2}\mathbf{i})^2=P^2\le(\mathbf{i}\circ\mathbf{R}\mathbf{i})\times(\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u}), \tag{4}$$

that is valid for periodic time vectors of the same dimension and non-singular matrix $\mathbf{R}$ which is symmetric with respect to the main diagonal.

The matrix $\mathbf{R}$ from (1) is symmetric with respect to the main diagonal and non-singular for at least two non-zero resistances of the transmission line, since $\det\mathbf{R}=r_Ar_B+r_Ar_C+r_Br_C$. The TWRF's vectors $\mathbf{u}=\begin{Vmatrix} u_{AC} & u_{BC} \end{Vmatrix}^{\wedge}$, $\mathbf{i}=\begin{Vmatrix} i_A & i_B \end{Vmatrix}^{\wedge}$ are periodic of the same dimension. Therefore, three important consequences for the theory of active filtering from formula (4) are valid for them.

First, from (4), we obtain the formula for apparent power as the maximum active power that can be achieved in the load for given voltages and power losses:

$$S=P_{MAX}=\sqrt{(\mathbf{i}\circ\mathbf{R}\mathbf{i})\times(\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u})}. \tag{5}$$

Secondly, the power loss in the transmission line cannot be less than the limit value determined from the expression

$$P_{LS}=\mathbf{i}\circ\mathbf{R}\mathbf{i}\ge P_{LS}^{MIN}=\frac{P^2}{\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u}}. \tag{6}$$

And, finally, thirdly, from the condition of vector $\mathbf{R}^{-1/2}\mathbf{u}(t)$ and $\mathbf{R}^{1/2}\mathbf{i}(t)$ proportionality, under which equality is achieved in (4), the active current formula follows

$$\mathbf{i}_A(t)=\frac{P}{\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u}}\mathbf{R}^{-1}\mathbf{u}(t), \tag{7}$$

which ensures the specified load power (2) at a unit value of the power factor $\lambda=P/S=1$.

Such an active current (7) causes power losses in the transmission line

$$\mathbf{i}_A\circ\mathbf{R}\mathbf{i}_A=\frac{P^2\times(\mathbf{R}^{-1}\mathbf{u}\circ\mathbf{R}\mathbf{R}^{-1}\mathbf{u})}{(\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u})^2}= \\ =\frac{P^2}{\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u}}=P_{LS}^{MIN}, \tag{8}$$

which coincides with the minimum value (6). Exactly this value must appear in the formula [13] to determine the power factor

$$\lambda=P/S=\sqrt{P_{LS}^{MIN}/P_{LS}}, \tag{9}$$

which corresponds to its physical content.

Then the energy-saving effect, which is estimated by the ratio of transmission line power losses in the absence and presence of the filter will be

$$W=\frac{P_{LS}}{P_{LS}^{MIN}}=\frac{\mathbf{i}\circ\mathbf{R}\mathbf{i}}{P^2/(\mathbf{u}\circ\mathbf{R}^{-1}\mathbf{u})}=\frac{S^2}{P^2}=\lambda^{-2}, \tag{10}$$

that coincides with [13].

Since the matrices $\mathbf{R},\mathbf{R}^{-1}$ in (5) and (7) can be determined with accuracy up to a constant factor, to simplify the implementation of the active current in the transmission line, we normalize these matrices relative to one of

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the resistances of the transmission line, for example, $r_A = r$. Then the other two resistances are characterized by the parameters $d = r / r_B ; q = r / r_C$, and the normalized matrices take shape

$$\bar{\mathbf{R}} = \mathbf{R} / r = \begin{Vmatrix} 1 & 0 \\ 0 & 1/d \end{Vmatrix} + \frac{1}{q} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}^{\wedge};$$

$$\bar{\mathbf{G}} = \bar{\mathbf{R}}^{-1} = \begin{Vmatrix} 1 & 0 \\ 0 & d \end{Vmatrix} - \frac{1}{1+d+q} \begin{Vmatrix} 1 \\ d \end{Vmatrix} \begin{Vmatrix} 1 \\ d \end{Vmatrix}^{\wedge}.$$

Next, we define the reference vector of the active current

$$\mathbf{u}_R(t) = \bar{\mathbf{G}}\mathbf{u}(t) = \left(\begin{Vmatrix} 1 & 0 \\ 0 & d \end{Vmatrix} - \frac{1}{1+d+q} \begin{Vmatrix} 1 \\ d \end{Vmatrix} \begin{Vmatrix} 1 \\ d \end{Vmatrix}^{\wedge} \right) \times$$

$$\times \begin{Vmatrix} u_{AC} \\ u_{BC} \end{Vmatrix} = \begin{Vmatrix} u_{AC} - u_0 \\ (u_{BC} - u_0)d \end{Vmatrix}; u_0 = \frac{u_{AC} + du_{BC}}{1+d+q}. \quad (11)$$

Taking into account this value (5), (6), (7) acquire their final form in TWRF

$$S = \sqrt{(\mathbf{i} \circ \bar{\mathbf{R}}\mathbf{i}) \times (\mathbf{u} \circ \mathbf{u}_R)}; \quad (12)$$

$$P_{LS}^{MIN} = \frac{P^2 r}{\mathbf{u} \circ \mathbf{u}_R}; \quad (13)$$

$$\mathbf{i}_A(t) = \frac{P}{\mathbf{u} \circ \mathbf{u}_R} \mathbf{u}_R(t). \quad (14)$$

Now we will show that (12) turns into the Buchholz's apparent power formula [21] in the form of the product of rms voltage and current values of a three-phase three-wire system

$$S_B = UI = \sqrt{T^{-1} \int_T [u_A^2(t) + u_B^2(t) + u_C^2(t)]dt} \times$$

$$\times \sqrt{T^{-1} \int_T [i_A^2(t) + i_B^2(t) + i_C^2(t)]dt},$$

if the resistances of the transmission line are equal, that is, for the special case of parameter values $d = q = 1$. Indeed, in this case it follows from (3) that $\mathbf{i} \circ \bar{\mathbf{R}}\mathbf{i} = T^{-1} \int_T [i_A^2(t) + i_B^2(t) + i_C^2(t)]dt.$ For voltages u_A, u_B, u_C, calculated relative to the point of artificial grounding [4], $u_A(t) + u_B(t) + u_C(t) = 0$, the following ratios are valid

$$u_0 = \frac{u_{AC} + u_{BC}}{3} = \frac{u_A + u_B - 2u_C}{3} = -u_C;$$

$$\mathbf{u}_R(t) = \begin{Vmatrix} u_{AC} - u_0 \\ u_{BC} - u_0 \end{Vmatrix} = \begin{Vmatrix} u_A \\ u_B \end{Vmatrix};$$

$$\mathbf{u}^{\wedge}\mathbf{u}_R = \begin{Vmatrix} u_{AC} \\ u_{BC} \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_A \\ u_B \end{Vmatrix} = \begin{Vmatrix} u_A \\ u_B \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_A \\ u_B \end{Vmatrix} - u_C \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_A \\ u_B \end{Vmatrix} =$$

$$= u_A^2 + u_B^2 + u_C^2;$$

$$\mathbf{u} \circ \mathbf{u}_R = T^{-1} \int_T [u_A^2(t) + u_B^2(t) + u_C^2(t)]dt,$$

which had to be proved.

Under the same condition of transmission line resistances equality (14) turns into the expression

$$\mathbf{i}_{AF}(t) = \frac{P}{T^{-1} \int_T [u_A^2(t) + u_B^2(t) + u_C^2(t)]dt} \begin{Vmatrix} u_A \\ u_B \end{Vmatrix},$$

which corresponds to the S. Fryze's formula [10] for the active currents of two phases of a three-wire power supply system. The correspondence of the third phase active current is ensured by the zero sum of the instantaneous values of the currents and voltages appearing in the Buchholtz's formula.

Formulas (12), (13), (14) lay the theoretical foundation for energy-efficient active filtering in TWRF with arbitrary ratios between transmission line resistances without the need to organize an artificial grounding point and with two instead of three SAF pulse regulators. As follows from (14) and the method of connecting SAF at PCC in fig. 1, the general strategy of its energy-efficient control in the TWRF consists in the formation of an active current in the transmission line and is implemented by the generating of two active filter currents

$$\mathbf{i}_F(t) = \begin{Vmatrix} i_{AF} \\ i_{BF} \end{Vmatrix} = \mathbf{i}(t) - \mathbf{i}_A(t) = \mathbf{i}(t) - \frac{\mathbf{u} \circ \mathbf{i}}{\mathbf{u} \circ \mathbf{u}_R} \mathbf{u}_R(t);$$

$$\mathbf{u}_R(t) = \begin{Vmatrix} u_{AC} - u_0 \\ d(u_{BC} - u_0) \end{Vmatrix}; u_0 = \frac{u_{AC} + du_{BC}}{1+d+q}. \quad (15)$$

Such control provides the minimum possible power losses in the transmission line according to (14) and requires two sensors of line voltages, two sensors of currents and the assignment of two parameters d and q that establish the ratio between the resistances of the transmission line.

3. *ACTIVE FILTERING STRATEGY THAT MINIMIZES POWER LOSSES IN A TRANSMISSION LINE WITH SINUSOIDAL AND BALANCED CONSUMED CURRENTS*

Modern standard IEEE Std. 1459-2010 describes apparent power as the maximal active power, which can be transmitted under sinusoidal and balanced conditions for voltage and current values. Therefore, the apparent power definition (12), in which periodic voltages and currents are of arbitrary shape, needs adjustment, but is useful as a basis for comparison.

Let the direct sequence detector [24], built on the second order generalized integrator for quadrature-signals generation [25], extract the voltage positive sequence component

$$\mathbf{u}_+(t) = \begin{Vmatrix} u_{AC+} & u_{BC+} \end{Vmatrix}^{\wedge} = \sqrt{2}U_+ \begin{Vmatrix} \cos(\omega t) & \cos(\omega t - \pi/3) \end{Vmatrix},$$

from the voltage vector of the general form $\mathbf{u}(t)$. Then the active current with sinusoidal and balanced components, which provides the active power P at the PCC, is determined by the expression

$$\mathbf{i}_{A+}(t) = \frac{P}{\mathbf{u} \circ \mathbf{u}_{R+}} \mathbf{u}_{R+}(t), \quad (16)$$

where $\mathbf{u}_{R+}(t) = \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix} = \begin{Vmatrix} u_{AC+} \\ u_{BC+} \end{Vmatrix} - \frac{u_{AC+} + u_{BC+}}{3} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}.$

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This current differs from the active current (14), which consists of asymmetric currents at different resistance values of the transmission line and causes power losses in the transmission line

$$P_{LS+} = \mathbf{i}_{A+} \circ \mathbf{R}\mathbf{i}_{A+} = \frac{P^2 \times (\mathbf{u}_{R+} \circ \mathbf{R}\mathbf{u}_{R+})}{(\mathbf{u} \circ \mathbf{u}_{R+})^2} = \frac{rP^2 (\mathbf{u}_{R+} \circ \bar{\mathbf{R}}\mathbf{u}_{R+})}{(\mathbf{u} \circ \mathbf{u}_{R+})^2}. \quad (17)$$

The obtained losses value differs from the minimum value according to (8). Let's find the ratio of power losses

$$k_{LS} = \frac{P_{LS}^{MIN}}{P_{LS+}} = \frac{P^2 r}{\mathbf{u} \circ \bar{\mathbf{G}}\mathbf{u}} \Bigg/ \frac{rP^2 (\mathbf{u}_{R+} \circ \bar{\mathbf{R}}\mathbf{u}_{R+})}{(\mathbf{u} \circ \mathbf{u}_{R+})^2} = \frac{(\mathbf{u} \circ \mathbf{u}_{R+})^2}{(\mathbf{u} \circ \bar{\mathbf{G}}\mathbf{u}) \times (\mathbf{u}_{R+} \circ \bar{\mathbf{R}}\mathbf{u}_{R+})}. \quad (18)$$

This ratio of power losses is determined exclusively by the voltage vectors $\mathbf{u}(t), \mathbf{u}_{R+}(t)$ and the resistance ratios of the transmission line, which is given by the matrices $\bar{\mathbf{R}}, \bar{\mathbf{G}}$ and does not exceed a unit. Its value can be used to predict the maximum thermal loading of three-wire transmission line in compliance with the existing requirements for the power quality at the PCC.

Let's find the value of this coefficient under the condition of a symmetrical sinusoidal three-phase source, when the voltages at the PCC are of the same form, i.e. $\mathbf{u}(t) = \mathbf{u}_+(t)$. In this case (18) takes the form

$$k_{LS+} = (\mathbf{u}_+ \circ \mathbf{u}_{R+})^2 / (\mathbf{u}_+ \circ \bar{\mathbf{G}}\mathbf{u}_+) \times (\mathbf{u}_{R+} \circ \bar{\mathbf{R}}\mathbf{u}_{R+}),$$

and individual components of this expression are calculated as follows:

$$\mathbf{u}_+^{\wedge}\mathbf{u}_{R+} = \begin{Vmatrix} u_{AC+} \\ u_{BC+} \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix} = \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix} - u_{C+} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix} = u_{A+}^2 + u_{B+}^2 + u_{C+}^2;$$

$$\mathbf{u}_+ \circ \mathbf{u}_{R+} = U_+^2.$$

$$\bar{\mathbf{R}}\mathbf{u}_{R+} = \left(\begin{Vmatrix} 1 & 0 \\ 0 & 1/d \end{Vmatrix} + \frac{1}{q} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}^{\wedge} \right) \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix} = \begin{Vmatrix} u_{A+} \\ u_{B+}/d \end{Vmatrix} - \frac{u_{C+}}{q} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix};$$

$$\mathbf{u}_+^{\wedge}\bar{\mathbf{R}}\mathbf{u}_+ = \begin{Vmatrix} u_{A+} \\ u_{B+} \end{Vmatrix}^{\wedge} \begin{Vmatrix} u_{A+} \\ u_{B+}/d \end{Vmatrix} - \frac{u_{C+}(u_{A+} + u_{B+})}{q} = u_{A+}^2 + u_{B+}^2/d + u_{C+}^2/q;$$

$$\mathbf{u}_+ \circ \bar{\mathbf{R}}\mathbf{u}_+ = \frac{U_+^2}{3}\left(1 + \frac{1}{d} + \frac{1}{q}\right) = \frac{U_+^2 (d + q + dq)}{3dq}.$$

To calculate the last component, we apply the formula for averaging the product of harmonic functions in the frequency domain [16]

$$\mathbf{u}_+ \circ \bar{\mathbf{G}}\mathbf{u}_+ = \mathrm{Re}(\bar{\mathbf{u}}_+^{\wedge}\bar{\mathbf{G}}\bar{\mathbf{u}}_+^*) = U_+^2 \,\mathrm{Re}\left[1 + d - \frac{(1 + de^{-j\pi/3})(1 + de^{j\pi/3})}{1 + d + q}\right] = U_+^2 \left(1 + d - \frac{1 + d + d^2}{1 + d + q}\right) = U_+^2 \frac{d + q + dq}{1 + d + q}.$$

Substitution of the obtained values finally gives

$$k_{LS+} = \frac{3dq(1 + d + q)}{(d + q + dq)^2}. \quad (19)$$

In connection with the difference between the minimum losses of the symmetrical sinusoidal mode and P_{LS}^{MIN}, according to the recommendations IEEE Std. 1459-2010 the determination of both the apparent power S_+ under such conditions and the power factor $\lambda_+ = P / S_+$ requires correction. Substitution P_{LS+} as the minimum achievable losses in (9) gives a modified formula for the power factor,

$$\lambda_+ = \frac{P}{S_+} = \sqrt{\frac{P_{LS+}}{P_{LS}}} = \sqrt{\frac{P_{LS}^{MIN}}{P_{LS}} \times \frac{P_{LS+}}{P_{LS}^{MIN}}} = \frac{\lambda}{\sqrt{k_{LS}}}. \quad (20)$$

which derives the modified formula for apparent power

$$S_+ = S\sqrt{k_{LS}}. \quad (21)$$

Thus, formulas (12), (13) retain their values for calculating extreme energy parameters of power systems, and in the related modified formulas (20), (21) the value k_{LS} according to (18) is a correction factor for calculating the apparent power and power factor when implementing restrictions on consumed currents to improve the power quality at PCC for the general case of transmission line resistance ratio.

The strategy of energy efficient SAF control with observance of the sinusoidal symmetrical form of the consumed currents consists in the formation of an active current (16) in the transmission line and is implemented by the current vector of the active filter

$$\mathbf{i}_{F+}(t) = \mathbf{i}(t) - \mathbf{i}_{A+}(t) = \mathbf{i}(t) - \frac{\mathbf{u} \circ \mathbf{i}}{\mathbf{u} \circ \mathbf{u}_{R+}} \mathbf{u}_{R+}(t);$$
$$\mathbf{u}_R(t) = \begin{Vmatrix} u_{AC+} - u_{0+} \\ u_{BC+} - u_{0+} \end{Vmatrix}; u_{0+} = \frac{u_{AC+} + u_{BC+}}{3}. \quad (22)$$

Such a strategy provides the minimum possible power losses in the transmission line (17) under such conditions and a gain in power loss

$$W_+ = \frac{P_{LS}}{P_{LS+}} = \frac{P_{LS}}{P_{LS}^{MIN}} \times \frac{P_{LS}^{MIN}}{P_{LS+}} = \frac{k_{LS}}{\lambda^2}. \quad (23)$$

And in this case, the apparent power (5) and the power factor (9) retain their value as bases for comparison when forming the consumed currents recommended by the actual standard.

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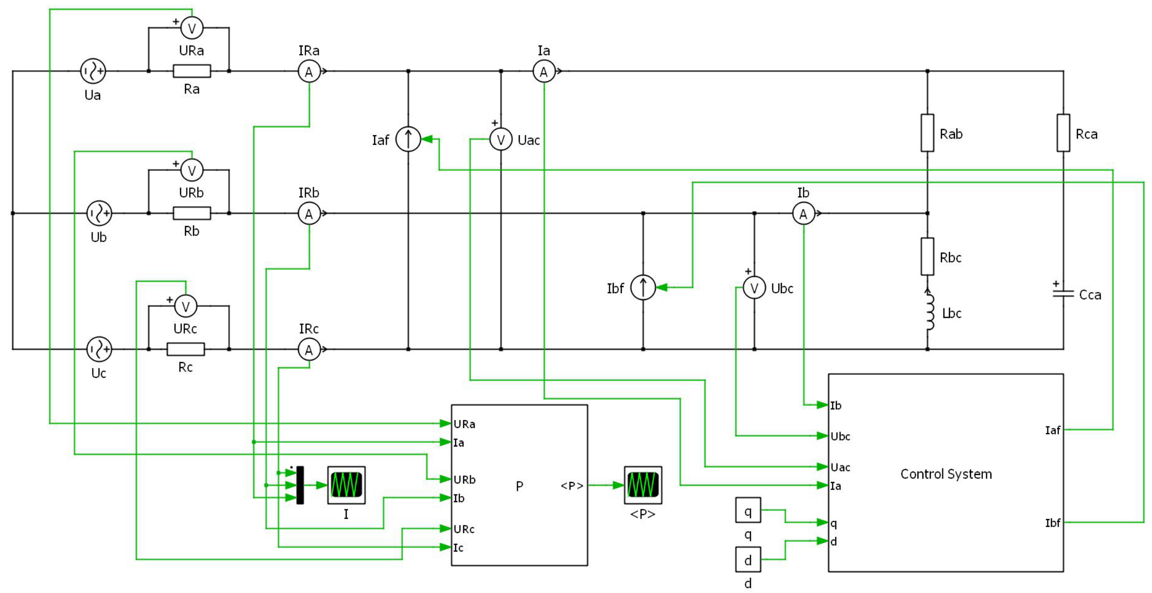

Fig. 2. Computer model with PAF of a three-phase three-wire power supply system and an asymmetric linear load

4. *Verification by computer simulation of the proposed energy efficient strategies*

For this, a computer model (Fig. 2) of a three-phase three-wire power supply system with an asymmetric linear load and SAF implemented by dependent current sources i_{AF} and i_{BF} with the Control System control module was created in the PLECS environment. The inputs of this module are supplied with the TWRF variables u_{AC}, u_{BC}, i_A, i_B from the corresponding sensors and the parameters *d*, *q* of the transmission line. The SAF strategy change is carried out by the Control System module forming the reference vector of the active current according to formulas (15) or (22) with the corresponding assignment of the *d* and *q* values. The P module calculates the average power losses in the transmission line for the period based on the currents and voltages of the transmission line resistors and displays the calculation results on the oscilloscope <P>. Another oscilloscope I displays the instantaneous values of the source currents.

Such model parameters were adopted: symmetrical sinusoidal voltages of a three-phase source with an rms value of the line voltage $U_+ = 100\sqrt{3}B$ and frequency $f = 50$ Hz, transmission line resistances are $r_A = r = 2m\Omega$; $r_B = \frac{r}{d} = 1m\Omega$ $(d = 2)$; $r_C = \frac{2}{q} m\Omega$, load resistances are $R_{AB} = 6\Omega$, $R_{BC} = 3\Omega$, $X_{BC} = 3\Omega \to L_{BC} = 9.54\,mH$, $R_{CA} = 4\Omega$, $X_{CA} = -5\Omega \to C_{CA} = 636.6\,\mu F$.

Table 1 presents the results of power losses simulation when applying different strategies of active filtering.

We obtain the theoretical dependence $P_{LS}^{MIN}(q)$, using (13) at $\mathbf{u} = \mathbf{u}_+; \mathbf{u}_R = \bar{\mathbf{G}}\mathbf{u}_+$. According to the results of the derivation of formula (19)

Table 1

Total power loss without filter				
q	0.5	1	2	4
P_{LS0}, W	12.4842	11.7340	11.3583	11.1703
Total power loss with a filter implementing the first strategy (15)				
q	0.5	1	2	4
P_{LS1}, W	11.1292	8.9064	6.9602	5.5694
Total power loss with a filter implementing the second strategy (22)				
q	0.5	1	2	4
P_{LS2}, W	12.9790	9.2773	7.4235	6.4959

$$\mathbf{u}_+ \circ \bar{\mathbf{G}}\mathbf{u}_+ = \frac{U_+^2(d+q+dq)}{1+d+q}.$$

The load active power the can be found by calculating the active conductivities of the linear loads G_{AB}, G_{BC}, G_{CA}:

$$P = U_+^2(G_{AB} + G_{BC} + G_{CA}) =$$

$$= U_+^2\left(\frac{1}{6} + \frac{3}{3^2+3^2} + \frac{4}{4^2+5^2}\right) = \frac{53U_+^2}{123}.$$

Substitution of the obtained values in (13) gives the analytical dependence of the minimum power loss on the ratio of transmission line resistances for the given parameters of the three-phase source and load

$$P_{LS}^{MIN}(d,q) = \frac{P^2 r}{\mathbf{u} \circ \mathbf{u}_R} = \left(\frac{53}{123}\right)^2 \times \frac{U_+^2 r(1+d+q)}{d+q+dq}.$$

Taking into account the numerical values of the model parameters, at $d = 2$, we finally obtain the desired analytical dependence

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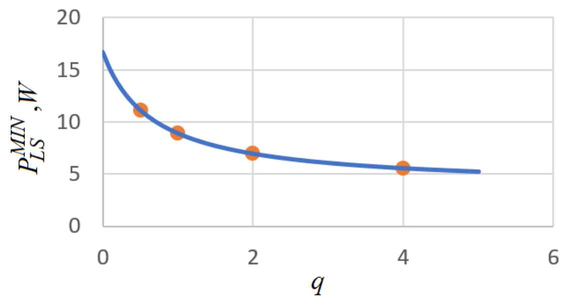

Fig. 3. Graph of dependence (24) with plotted points of experimental losses when using the first filtering strategy

$$P_{LS}^{MIN}(q) = 60 \times \left(\frac{53}{123}\right)^2 \times \frac{3+q}{2+3q}. \qquad (24)$$

Graph of dependence (24) with plotted points of experimental losses P_{LS1} from table 1 is shown in fig. 3. We note a complete coincidence of the results of computer modeling with the theoretical dependence of the minimum power loss on the parameter of the transmission line, which confirms the adequacy of the proposed strategy (15) of SAF control in TWRF.

In order to confirm the adequacy of the second proposed strategy (22), we will carry out an experimental verification of the theoretical dependence of the power losses factor (19), which at $d = 2$ takes the form

$$k_{LS+}(q) = \frac{6q(3+q)}{(2+3q)^2}. \qquad (25)$$

The graph of analytical dependence (25), presented in Fig. 4, is fully confirmed by the 4 point values of the fractions P_{LS1} / P_{LS2} calculated on the data of table 1 at the corresponding values of the argument plotted on it. This verifies the second superimposed strategy (22), which minimizes power losses while ensuring the symmetry of consumed currents.

To confirm the last statement, a sequential change in filtering strategies was organized over time while observing the consumed currents and loss levels (time diagrams in Fig. 5) when $d = 2, q = 4$. The first time interval ($t \in [0, 0.1]$) corresponds to the absence of filtering, while the asymmetry of the consumed currents is maximal, and

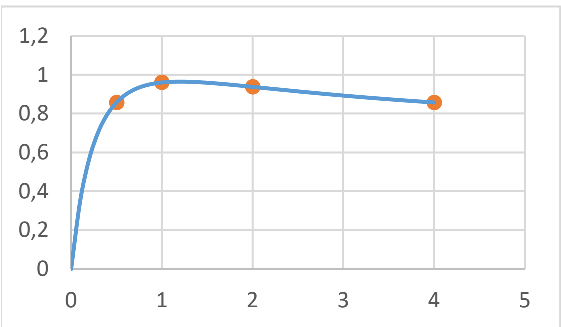

Fig. 4. Graph of dependence (25) with plotted points of experimental fractions P_{LS1} / P_{LS2} from Table 1

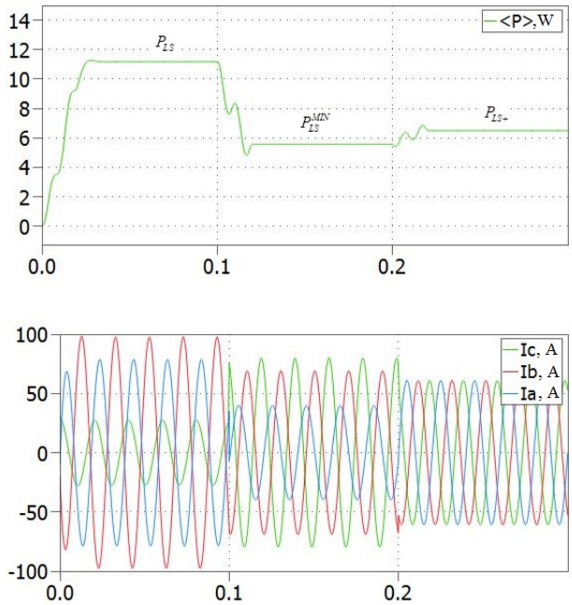

Fig. 5 Time diagrams of power losses and consumed currents under different SAF control strategies

the power loss P_{LS} is also maximal. Filtering according to the first strategy ($t \in [0.1, 0.2]$) gives the minimum loss level P_{LS}^{MIN}, which is confirmed both by the theoretical dependence (24) at $d = 2, q = 4$ and by the table values of the experiment, but the asymmetry of the consumed currents is still significant. And only filtering according to the second strategy ($t \in [0.2, 0.3]$) provides a symmetrical sinusoidal mode of consumption with a slight increase in the loss level P_{LS+} in strict accordance with the experimentally and theoretically confirmed value of the coefficient $k_{LS+}(d = 2, q = 4) = 6/7$. At the moment of switching the type of filtering, the currents change their phase and amplitude, and transient processes are observed on the oscillogram of the average power loss.

Thus, the complete coincidence of the computer modeling results with the predicted analytical dependences of the power losses and the shapes of the curves of the consumed currents confirms the adequacy of both the proposed SAF control strategies in the TWRF, and the correctness of the model functioning.

Conclusions

The basic concepts of the integral power theory of a three-phase three-wire system, such as apparent power, active current and minimum power loss in the transmission line, are defined for the general case of different resistances of the transmission line in the reduced coordinate basis of the two-wattmeter method, which simplifies the realization of energy-efficient SAF control strategies.

The strategy of active filtering in TWRF, which minimizes the power losses of a three-phase three-wire transmission line with different values of its resistances, was developed and verified by a virtual experiment.

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The strategy of active filtering in TWRF was developed and verified by a virtual experiment to improve the power quality at the points of common coupling, which ensures the minimum power loss of the transmission line with symmetrical sinusoidal currents of a three-phase source.

The correction factor for apparent power and power loss gain formulas in the presence of restrictions on consumed currents was determined and verified by a virtual experiment. Its value can be used to predict the maximum thermal loading of a three-phase three-wire transmission line in compliance with the existing requirements for the power quality.

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***Анотація*—Напівпровідникові паралельні активні фільтри (ПАФ) є ефективним засобом підвищення якості електроенергії шляхом компенсації неактивних складових потужностей на шинах трансформаторних підстанцій трифазних ліній електропередачі 6/0,4 кВ. Відомі стратегії керування ПАФ оперують з лінійно залежними трикоординатними векторами лінійних струмів та фазних напруг, що відраховуються від точки штучного заземлення, що породжує невиправдану складність системи керування ПАФ через надмірну кількість сенсорів і регуляторів та необхідності організації точки штучного заземлення. Метою статті є розроблення енергоефективних стратегій паралельної активної фільтрації в системі координат методу двох ватметрів (СКМДВ) з безпосереднім використанням двокоординатних векторів напруг та струмів. Для цього такі базові поняття теорії інтегральної потужності трифазної трипровідної системи, як повна потужність, активний струм та мінімальна потужність втрат в лінії передачі були виражені в скороченому координатному базисі методу двох ватметрів. На основі цих понять розроблені та верифіковані віртуальними експериментами дві енергоефективні стратегії активної фільтрації в СКМДВ. Перша з них мінімізує потужність втрат трифазної трипровідної лінії передачі з різними значеннями її опорів та забезпечує одиничне значення коефіцієнта потужності. Друга стратегія відповідно до рекомендації IEEE Std. 1459-2010 забезпечує мінімальну потужність втрат лінії передачі при симетричних синусоїдних струмах трифазного джерела. Визначений та верифікований віртуальним експериментом коригувальний коефіцієнт формул повної потужності та виграшу за потужністю втрат за наявності обмежень на симетричну синусоїдну форму споживаних струмів, значення якого може бути використане для прогнозування максимального теплового навантаження трифазної трипровідної системи з дотриманням існуючих вимог на якість електричної енергії в точках загального підключення.**

Ключові слова — трифазна трипровідна система живлення; повна потужність; активний струм; енергоефективні стратегії активної фільтрації; система координат методу двох ватметрів.