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Prediction of the Time Distribution of Shannon and Renyi Entropy Based on the Theory of Moments

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Abstract—The application of the theory of moments to distributed generation systems for the construction of a reducing and predicting polynomial of the time distribution of entropy changes in time at the base interval is proposed. It is shown that in order to improve the accuracy of forecasting, it is necessary to take into account the fractal nature of energy consumption processes and use Rényi entropy in calculations. By taking into account the fractal nature of the energy consumption process and the use of Rényi entropy in calculations, an increase in prediction accuracy by 11% is achieved, resulting in the prediction of the time distribution of Shannon's entropy for power consumption with an error not exceeding 23%.

Keywords — distributed generation systems; Shannon entropy; Rényi entropy; fractal dimension.

I. INTRODUCTION

When considering the issue of efficient use of primary energy in distributed generation systems [1-3], it should be taken into account that in accordance with the Law of Ukraine "On Amendments to Certain Laws of Ukraine on Improving the Conditions for Supporting the Production of Electricity from Alternative Energy Sources", from 2022, full responsibility for the imbalance of actual and forecasted electricity production schedules is introduced for all producers with installations based on renewable energy sources [4]. This creates economic incentives to increase the accuracy of forecasting electricity generation and consumption schedules and contributes to the development of the balancing capacity sector.

The states into which the distributed generation system, as a complex statistical open macrosystem, transitions due to changes in the states of devices for converting electrical energy parameters are unequally probable [5-9]. The main reason for the heterogeneity in probability is the presence of several parallel existing distributions of energy generation and consumption [10]. For distributed generation systems, the role of the function that uniquely reflects the probability of the macrostate is performed by the entropy divergence [11], which takes into account not only the uniform but also the arbitrary nature of the prior probability distribution. Numerically, the entropy divergence is equal to the statistical distance between the current and maximum permissible distributions of energy flows of generation and consumption and is defined as the difference of the corresponding entropies. Therefore, one of the main current tasks that arise when studying the operating modes of distributed generation systems is the implementation of predictive control for entropy divergence based on the prediction of the time distribution function of the entropy of the power consumption.

II. CONSTRUCTION OF A FORECAST EQUATION OF CONSUMPTION POWER BASED ON INSTANTANEOUS TRANSFORMATIONS

Taking into account the analog of the Heisenberg uncertainty principle in dispersed generation systems [12] indicates the need to implement a two-channel control system for efficient use of energy in systems: 1) by the maximum duration of the base interval to provide the required level of energy for charging the drive; 2) by the minimum duration of the observation interval to ensure the required level of the maximum possible energy obtained from installations based on renewable sources. To ensure the efficient use of primary energy in dispersed generation systems, the installed capacity of the electrical energy storage device must be sufficient both to provide an average value of power consumption and to balance power at peak intervals consumption, which requires an increase in the accuracy of both short-term (at observation intervals Δt_j) and long-term (at

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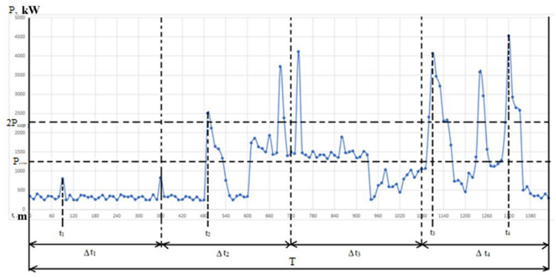

Fig.1. Typical schedule of electricity consumption in a private house located in the Netherlands for May 13, 2010 [13]

the baseline interval T) forecasting of consumption capacity, which is shown in Fig. 1.

In various problems of the theory of random processes, the theory of automatic regulation, and transformative technology, the theory of moments induced by the system is used for the problem of predicting time series, $\left\{t^k\right\}_0^\infty$ which are parameters of spectral characteristics of signals [14]. Considering the widespread use and traditionality of moments induced by the system, as well as the unevenness of graphs of power consumption, we will consider the possibility of applying the theory of moments to predict entropy consumption capacity in systems with installations based on renewable energy sources.

Moments, according to Stiles [15] at the *i*-th observation interval Δt_i (see Fig. 1) are defined as functionals:

$$m_k^i = \int\limits_0^\infty t^k d\sigma(t), \qquad (1)$$

where $k = 0, 1, 2, \ldots$, m_k^i is the moment of *the k-th* order for the function $\sigma(t)$, $d\sigma(t) = \sigma'(t)dt = f(t)$. In the classical problem of moments, it is necessary to find a function $\sigma(t)$ that is defined by a sequence of numbers m_k^j, $k = 0, 1, 2, \ldots$ if the function $\sigma(t)$ is discrete, then the formula for calculating moments is as follows:

$$m_k^i = \sum_{i=0}^{\infty} (i\tau)^n f(i\tau), \qquad (2)$$

where $i \in (0 \ldots \infty)$ is the reference number.

Applying the moment transformation to restore probabilistic processes of energy consumption in dispersed generation systems at the base interval $T = r \cdot \Delta t$ (see Fig. 1) and considering $f(i\tau)$ Shannon's entropy as a function, we get an expression describing the time distribution of probabilistic moments $m_k^T\{f(p)\}$:

$$m_k^T = T\sum_{j=0}^{r} (jT)^k \times \left(-\sum_{i=0}^{q} p^n(i\tau, jT)\ln p(i\tau, jT)\right), \qquad (3)$$

where $k = 0,1,2,\ldots$, $f(p) = \ln p$ is the function of the discrete argument; $p(i\tau)$ – distribution of the probability of the process of energy consumption in time at each of the i observation intervals Δt; q – the number of power consumption measurements at each observation interval Δt; $m_0 = p\ln p$ is a probabilistic moment of zero order, corresponding to Shannon's informational entropy. That is, Shannon entropy is used to restore energy consumption processes in dispersed generation systems at the base interval T.

To restore $f(p)$ the argument p function by its vector $m_k^T\{f(p)\}$, the inverse moment transformation is used $\Theta_N^{-1}\{\vec{m}_N\}$:

$$f(p) \approx \sum_{k=0}^{K} c_k \varphi_k(p), \qquad (4)$$

where $K \le N$, c_k are the corresponding numerical coefficients, $\varphi_k(p) \in \{\varphi_k(p)\}_O^K$ are the basic functions satisfying the condition

$\underset{k=0,K}{\forall} \varphi_k(p), p \in (0,1) \underset{n=0,N}{\exists} \left|m_n\{\varphi_k(p)\}\right| < \infty$ [14] considers the conditions that the system of reducing functions must meet, and shows that these conditions correspond to the orthogonal system of Legendre polynomials, the elements of which are calculated in accordance with the expression [16]:

$$P_k(p) = \left(\frac{1}{k!}\right)\frac{d^k}{d(p)^k}\left(p^2 - p\right), \qquad (5)$$

where $k = 0,1,2\ldots$, $p \in (0,1)$, $P_k(p)$ is the k element of the system of Legendre polynomials, reduced to the interval (0,1). Note that multiplying each element $P_k(p)$ by $1/\sqrt{2k+1}$, we get an orthogonal system $\{P_k'(p)\}_0^\infty, P_k'(p) = P_k(p)/\sqrt{2k+1}$, the elements of which for k = 0, 1, 2, 3, 4 are given in Table 1, and the matrix of images W_4^k – in Table 2.

TABLE 1 LEGANDRE POLYNOMIALS

Element of the system	Equation
$P_0'(p)$	1
$P_1'(p)$	$(2p-1)/\sqrt{3}$
$P_2'(p)$	$(6p^2-6p+1)/\sqrt{5}$
$P_3'(p)$	$(20p^3-30p^2+12p-1)/\sqrt{7}$
$P_4'(p)$	$(70p^4-140p^3+90p^2-20p+1)/\sqrt{9}$

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TABLE 2 MATRIX OF IMAGES

1	0	0	0	0
$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	0	0
$\frac{1}{3}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6\sqrt{5}}$	0	0
$\frac{1}{4}$	$\frac{3\sqrt{3}}{20}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{20\sqrt{7}}$	0
$\frac{1}{5}$	$\frac{2}{5\sqrt{3}}$	$\frac{2}{7\sqrt{5}}$	$\frac{1}{10\sqrt{7}}$	$\frac{1}{70\sqrt{9}}$

The given image matrix allows you to reconstruct a function describing the change in Shannon entropy $f(p)$ along the fourth-order momentum vector $m_4\{f(p)\}$ using formula (4), where $\varphi_k(p') = P_k'(p)$.

Restoration of the time dependence of moments at observation intervals for the entire base interval allows you to forecast a certain base interval to ensure the efficient use of primary energy by correcting the storage capacity.

Demonstration of the described approach will be considered as a specific example of entropy distribution forecasting. As initial data for calculations, data on the power consumption of a private house obtained from 2007 to 2010 with discreteness of 10 minutes are used [13]. The base interval is equal to a day. To calculate probabilistic moments, it is necessary to obtain a probability distribution p_i process of consumption in time, for which the probability density is estimated in the form of a histogram according to the following algorithm:

1. The minimum $x_{\min}$ and maximum $x_{\max}$ elements of the sample implementation are determined.
2. The range of variations is determined $\Delta = x_{\max} - x_{\min}$.
3. The number of histogram intervals is determined according to the Sturgess formula [17] $K \approx 1 + 3{,}32\lg N$, where N is the sample size.
4. The histogram step is determined $\Delta x = \frac{\Delta}{K}$.
5. The frequencies of the elements of the $N(\Delta x)$ sample implementation in each of the histogram intervals are calculated.
6. For each histogram interval, columns are drawn with a height of $\hat{p}(x) = \frac{N(\Delta x)}{N \cdot \Delta x}$.

Then the probability of the element of the sample implementation falling x_i into the corresponding histogram interval is determined by the formula:

$$p(x_i) = \frac{N_i(\Delta x)}{N}, \quad (6)$$

and Shannon's entropy according to the formula:

$$H = -\sum_i p(x_i)\ln p(x_i), \quad (7)$$

where the entropy values are calculated at the end of each hour.

At the same time, the probabilities $p(x_i)$ of the element of the sample implementation falling x_i into the corresponding histogram interval have a logarithmic dependence on the sample size N, and the Sturgess formula determines the number of quantization levels.

The results of the calculation of probability values, Shannon entropy, and probability density histogram for May 13, 2010, are given in Table 3.

Probability density histogram is shown in Fig. 2.

TABLE 3 RESULTS OF THE CALCULATION

Time interval	Time	Power, kW	Probability	Shannon entropy	Renyi entropy
0	00:00	348,046	0,625		
1	00:10	270	0,625		
2	00:20	402,448	0,625		
3	00:30	325,84	0,625		
4	00:40	250	0,625		
…	…	…	…	…	…
139	23:10	350,685	0,041667		
140	23:20	362,988	0,041667		
141	23:30	298	0,041667		
142	23:40	401,462	0,041667		
143	23:50	294	0,041667	1,763	5,0405

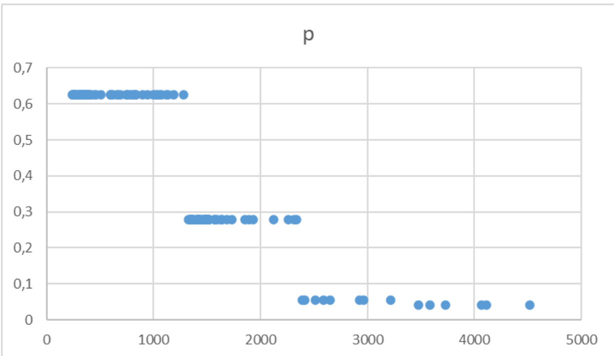

Fig.2 Probability density histogram

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TABLE 4 CALCULATED VALUES OF PROBABILISTIC SHANNON

Moment	Value
m_0^T	40,986
m_1^T	484,088
m_2^T	7433,809
m_3^T	128663,896
m_4^T	2384076,562

The values of the first five probabilistic moments m_0^T, m_1^T, m_2^T, m_3^T, m_4^T, calculated on the base interval according to formula 4 are given in Table. 4.

The technique for restoring the entropy distribution using the Legendre system of polynomials is as follows.

1. The numerical coefficients of the series are calculated c_k by multiplying the matrix of images W_4^k by the vector of moments $m_k^T\{f(p)\}$:

$$c_k=\left(W_4^k\right)^{-1}\cdot\left(m_k^T\right)^T=\begin{pmatrix}40{,}986\\ 1{,}606\cdot10^3\\ 9{,}333\cdot10^4\\ 6{,}233\cdot10^6\\ 4{,}486\cdot10^8\end{pmatrix}.$$

2. The found vector of numerical coefficients is multiplied by the Legendre system of polynomials, and the reducing polynomial is obtained:

$$\begin{gathered}f(p)=1{,}1116\cdot10^{10}\cdot p^4-2{,}2183\cdot10^{10}\cdot p^3+\\ +1{,}4217448\cdot10^{10}\cdot p^2-\\ -3{,}146336\cdot10^9\cdot p+1{,}56349\cdot10^8\end{gathered}.$$

3. The restored temporal distribution of entropy is obtained by substituting the values of probabilities for the selected day into the resulting polynomial.

Fig. 3 shows the calculated from real data (blue curves), restored (orange curve) and predicted (green curve) time distributions of Shannon entropy for power consumption for two days: May 13 and 14, 2010. Forecasting for the next day uses power data for the previous day.

The mean absolute percentage error (MAPE) of the recovery of the time distribution of Shannon's entropy using moment transformations, calculated by the formula [18]:

$$MAPE=\frac{1}{N}\sum_{i=1}^{N}\frac{\left|H_i-\bar{H}_i\right|}{\bar{H}_i}\cdot100\%,\qquad(8)$$

where H_i and $\bar{H}_i$ are the real and restored values of entropy, respectively, for the example under consideration,

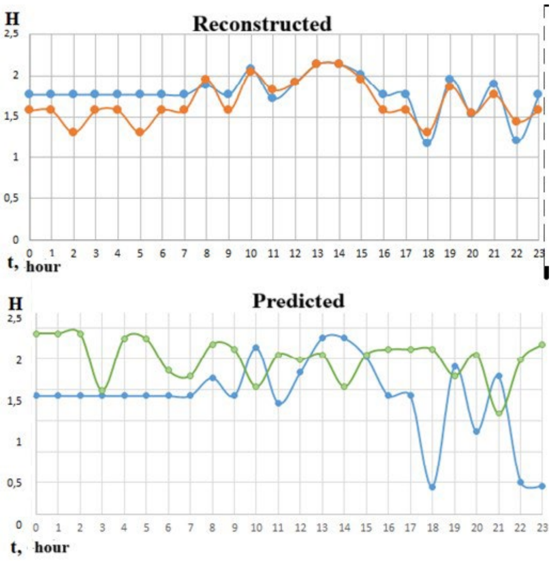

Fig. 3. Time distributions of Shannon's entropy: calculated (blue curves); restored (orange curve); predicted (green curve)

is 9.08%, and the average absolute percentage error of forecasting is 22.53%.

Thus, the error of reconstructing the time distribution of Shannon entropy for consumption power using the theory of moments does not exceed 10%, which allows us to use the described approach to predicting the temporal distribution of entropy of consumption power in dispersed generation systems.

III. TAKING INTO ACCOUNT THE FRACTAL NATURE OF THE ENERGY CONSUMPTION PROCESS

Studies have shown that improving the accuracy of forecasting power consumption is achieved by considering the fractal nature of the energy consumption process [19], which requires moving from the calculation of Shannon's entropy to the calculation of Rényi's entropy when predicting power consumption.

Given the fractal dimension of the power consumption curve, the Rényi entropy values are calculated using the formula:

$$R(D)=\frac{1}{1-D}\ln\left(\sum_{i=1}^{N(\varepsilon)}p_i^D\right),\qquad(9)$$

where $D=\lim_{\varepsilon\to0}\log n(\varepsilon)/\log\left(\frac{1}{\varepsilon}\right)$ is the fractal dimension of the studied power consumption curve; ε is the size of the unit cell, and is $n(\varepsilon)$ the number of such cells required to cover the power change curve.

The calculation of the fractal dimension is carried out by the cell method [20], the essence of which is that when calculating the fractal dimension, instead of balls, a set of rectangular grids with different cell sizes is used, mainly in the shape of a square. For each grid, the number of cells covering at least one point of the broken line

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of the electricity consumption graph is calculated $n(\varepsilon_i)$ from the size of the cell ε_i , where i = 1, 2, ... corresponds to the nature of the mesh in this set. Next, for this dependence, a graph of the function is plotted $\log n(\varepsilon_i) = f(\log(\varepsilon_i))$ on a logarithmic scale. The points marked on the graph are approximated by a straight line using the method of least squares. The resulting equation of the line has the form:

$$\log y = -k \log x + b, \quad (10)$$

where x and y are logarithmic coordinates, and the positive value of the parameter k corresponds to the value of the fractal dimension D of the time series of power consumption.

Fig. 4, a – d shows an example of superimposing a grid with cells of different sizes on the graph of changes in power consumption for May 13, 2010. The initial minimum cell size is chosen as not less than the smallest distance between two adjacent numerical values of the time series. The minimum difference between the two values for the selected date is 68. Then we find the cell size – for this, we convert the hours into minutes. As a result, we have the time value in minutes from 0 to 1440 minutes along the abscissa axis and the value of power consumption along the ordinate axis. Then, along the ordinate axis, select a size not less than 68, and in accordance with the abscissa axis, we find the size along the abscissa axis, to eventually get a grid in the form of a cell. As a result, we have the cell size along the ordinate axis ε_y = 100, and along the abscissa axis ε_x = 10. Next, we select the value of the step along the abscissa axis, where it is worth choosing by the number of levels covered by the grid cells, that is, the number of values falling into one level. ε_y ε_x to get two points to the same level.

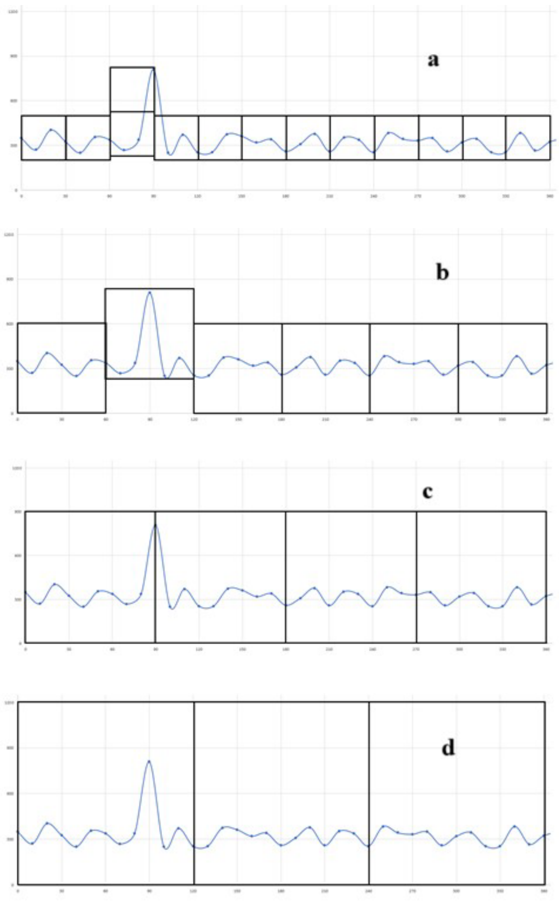

Fig. 4. An example of overlaying on the graph the change in power consumption of a grid with cells of different sizes: a) =30; b) =60; c) =90; d) =120

Table. 5 shows the results of calculating the number of cells for each step (ε_y = 30, 60, 90, 120, 150) and the total number of cells, as well as the logarithms of these values.

TABLE 5 CELL COUNT VALUE FOR EACH CELL RESIZING STEP

Cell size		Number of cells	
ε_i	$\log(\varepsilon_i)$	$n(\varepsilon_i)$	$\log(n(\varepsilon_i))$
30	5,704	139	4,935
60	6,397	59	4,078
90	6,802	40	3,689
120	7,09	26	3,258
150	7,313	19	2,944

Fig. 5 shows in logarithmic coordinates a graph of the dependence of the number of cells $\log(n(\varepsilon_i))$ on their size $\log(\varepsilon_i)$ for the time series of changes in consumption power for May 13, 2010 and its approximation by a straight line using the method of least squares.

The equation of approximation of the graph of the dependence of the number of cells on their size is as follows:

$$y = -1{,}2166 \cdot x + 11{,}885,$$

where the positive angular coefficient corresponds to the value of the fractal dimension, and the high quality of the approximation is confirmed by the value of the coefficient of determination close to one R^2 = 0.9963.

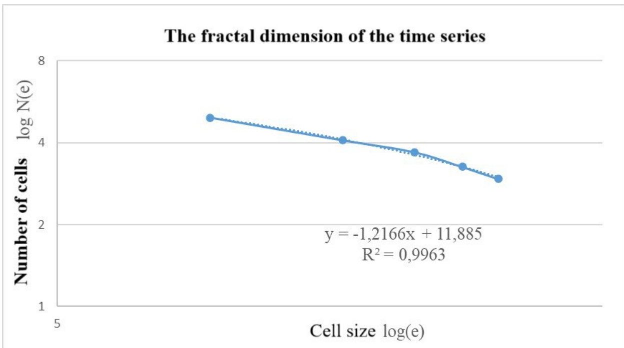

Fig. 5. Graph of the dependence of the number of cells on their size in the logarithmic coordinate system

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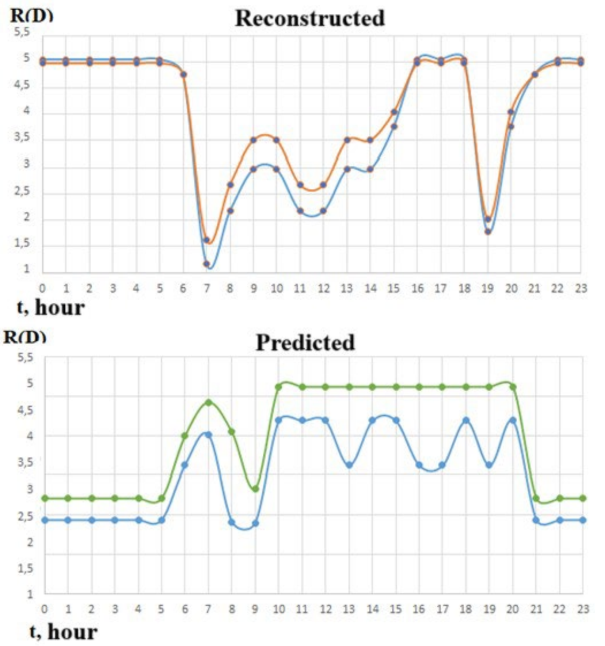

Fig. 6. Time distributions of Renyi entropy: calculated (blue curves); restored (orange curve); predicted (green curve)

Reducing the polynomial of the Rényi entropy distribution using probabilistic moments calculated by the formula:

$$m_k^T = T\sum_{j=0}^{r}(jT)^k \times \frac{1}{1-D}\left(\ln\sum_{i=0}^{q}p^D(i\tau, jT)\right), \qquad (11)$$

It looks like this:

$$f(p) = 2,7206666\cdot 10^{10}\cdot p^4 - 5,4295935\cdot 10^{10}\cdot p^3 + \\ +3,480449\cdot 10^{10}\cdot p^2 - \\ -7,70347969\cdot 10^9\cdot p + 3,828930223\cdot 10^8$$

Substituting the values of probabilities into the obtained polynomial, we get the restored and predicted time distributions of Rényi's entropy in Fig. 6 for two days: May 13 and 14, 2010.

The mean absolute percentage error of reconstructing the time distribution of Rényi's entropy using moment transformations for the example under consideration is 8.35%, and the percentage error of forecasting is 20.12%, which increases the accuracy of the forecast compared to the use of Shannon entropy.

When using traditional approaches to predicting the temporal distribution of entropy of Shannon and Rényi using neural networks, the prediction errors do not exceed 15%, which allows us to use the described approach to predicting the temporal distribution of entropy of power consumption based on the theory of moments in dispersed generation systems. The disadvantage of neural network-based prediction models is the need to train the model on significant amounts of training data. Model training also requires the correct selection of learning parameters, such as the number of learning epochs, the initial degree of learning, the number of cells in the neural network, etc. Although the forecasting method proposed in the article has a slightly higher error of 20%, it is much easier to apply and can operate with less training data.

Conclusion

Applying the theory of moments to construct a reducing and predictive polynomial allows, with an error not exceeding 23%, to predict the time distribution of Shannon's entropy for power consumption. An increase in prediction accuracy of 11% is achieved by considering the fractal nature of the energy consumption process and the use of Rényi entropy in calculations.

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Прогнозування часового розподілу ентропії Шеннона та Реньї на основі теорії моментів

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***Анотація*—У цій статті запропоновано підхід до прогнозування часового розподілу ентропії Шеннона та Реньї на основі застосування теорії моментів. Запропоновано застосування імовірнісних моментів для побудови відновлюючого та прогнозуючого поліному часового розподілу зміни ентропії на базовому інтервалі. Показано, що для покращення точності прогнозування необхідно врахувати фрактальну природу процесів споживання енергії та при розрахунках використовувати ентропію Реньї. Показано, що однією з основних актуальних задач, яка виникає при дослідженні режимів роботи систем розосередженої генерації є реалізація упереджувального керування за ентропійною дивергенцією на основі прогнозування функції часового розподілу ентропії потужності споживання. Для забезпечення ефективного використання первинної енергії у системах розосередженої генерації необхідно, щоб встановлена ємність накопичувача електричної енергії була достатньою як для забезпечення середнього значення потужності споживання, так і для балансування потужності на інтервалах пікового споживання, що потребує підвищення точності як короткострокового (на інтервалах спостереження) так і довгострокового (на базовому інтервалі) прогнозування потужності споживання. Наведено вираз, що описує часовий розподіл імовірнісних моментів на базовому інтервалі, розглядаючи в якості функції $f(i\tau)$ ентропію Шеннона. Наведено формулу зворотного моментного перетворення. Наведено вираз, відповідно до якого розраховуються елементи ортогональної системи поліномів Лежандра, які використовуються в якості системи відновлюючих функцій. Наведено матрицю зображень, що дозволяє відновити функцію, що описує зміну ентропії Шеннона за вектором моментів четвертого порядку. Відновлення часової залежності моментів на інтервалах спостереження для всього базового інтервалу дозволяє виконати прогнозування на деякий базовий інтервал для забезпечення ефективного використання первинної енергії за рахунок корекції ємності накопичувача. Розглянуто демонстрацію описаного підходу на конкретному прикладі прогнозування часового розподілу ентропії. Для розрахунку імовірнісних моментів, необхідно отримати розподіл імовірностей процесу споживання у часі, для чого наведено алгоритм оцінки щільності імовірності у вигляді гістограми. Описано методику відновлення розподілу ентропії з використанням системи поліномів Лежандра. Наведено часові розподіли ентропії Шеннона для потужності споживання для двох діб. Покращення точності прогнозування потужності споживання досягається шляхом врахування фрактальної природи процесу споживання енергії, що потребує перейти від розрахунку ентропії Шеннона до розрахунку ентропії Реньї. Описано суть клітинкового методу, за яким відбувається розрахунок фрактальної розмірності кривої потужності споживання. Наведено формулу для розрахунку відновлюючого поліному розподілу ентропії Реньї з використанням імовірнісних моментів. . Наведено часові розподіли ентропії Реньї для потужності споживання для двох діб. Отримана точність прогнозування дозволяє використовувати описаний підхід до прогнозування часового розподілу ентропії потужності споживання на основі теорії моментів у системах розосередженої генерації.**

Ключові слова* — *системи розосередженої генерації; ентропія Шеннона; ентропія Реньї; фрактальна розмірність.