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Selection of scales for continuous wavelet transform for improving patterns detectability

Рассмотрена задача выбора масштабных коэффициентов для непрерывного вейвлет-анализа. Предложен способ вычисления максимального и минимального требуемых масштабов для временной локализации паттернов в сигнале. Использование результатов работы проиллюстрировано на примере подбора масштабных коэффициентов для решения задачи временной локализации эпилептиформных комплексов в электроэнцефалограмме.

The task of scales selection for continuous wavelet transform is analyzed in brief. The technique of computation the minimal and maximal required scales for better pattern localization in a signal is proposed. The proposed technique is employed for time localization of epileptiform patterns in the electroencephalogram.

Introduction

The different types of wavelet-transforms (WT) recently gained widening in the field of signal analysis, coding, adaptive filtration and compression of one- and multidimensional signals. It could be explained by inherent flexibility in application of WT i.e. ability of selecting the mother wavelet from wide diversity of functions with different properties. One of the most useful advantages of WT is the possibility to explore time behaviour of a signal under consideration by analyzing the results of WT for different time shifts. Thus the scale-dependent signal analysis could be performed for arbitrary time intervals with high resolution in time and frequency.

The basic parameters for WT calculations are the scale factors for stretching and dilating of mother wavelet function and also a step and range of time shifts for moving a scaled function along the time interval on which the signal is analyzed. In the analysis of biomedical signals the use of continuous WT (CWT) is the most widespread as it enables to use the redundancy which is inherent to wavelet representation of a signal for the thorough time-scale analysis. But the other side of such advantage is large amount of calculations to be performed for obtaining the coefficients of wavelet signal decomposition that requires considerable computational resources. Preliminary selection of scales at which the calculations of CWT coefficients are conducted for will shorten the calculation procedure and will potentially decrease total time consumption for signal analysis. In addition, the selection of scales for CWT can help to draw researcher’s attention only on those scales, where essential properties of analyzed signal will manifest.

The purpose of this work is the development of one possible approach to preselect the scale factors interval for mother wavelet function. A case when CWT is used for time localization of patterns with known time properties is considered.

1. Continuous wavelet transform for the analysis of discrete signals

Theory of wavelet transform is sufficiently expounded elsewhere [1, 2]. In this work the attention will be limited to consideration of continuous wavelet transform only, as other kinds of wavelet transformation techniques (frame decompositions, dyadic orthogonal, wavelet-packages) could be derived from CWT with imposition of additional requirements and limitations whereas the CWT could be considered as the generic approach for any wavelet-derived signal decompositions.

Continuous wavelet transform of continuous signal $f(t) \in L^2(R)$ transforms the signal into two-dimensional space of wavelet coefficients $Wf_\Psi(a,b) \in L^2(R/0 \times R)$ and can be regarded as inner product of a signal and a scaled wavelet

$$W(f)_\Psi(a,b) \equiv \int_{-\infty}^{\infty} f(t) \cdot \psi^*_{a,b}(t)dt, \qquad (1)$$

where an asterisk denotes the complex conjugate of wavelet function $\psi_{a,b}(t)$ which is obtained from an initial «mother» wavelet function $\Psi(t)$ with the use of scale coefficients a $(a \in R\ a \neq 0)$ and shift parameters b:

$$\psi_{a,b}(t) \equiv |a|^{-\frac{1}{2}} \Psi\left(\frac{t-b}{a}\right). \qquad (2)$$

From (1) is evident that CWT of signal is the decomposition of the signal into components obtained by all possible compressions, dilations and time displacements of one mother wavelet function $\Psi(t)$.

According to Calderon, Grossmann and Morlet theorem [2] the coefficients (1) will represent the initial signal $f(t)$ only when the admissibility condition holds for mother wavelet function:

$$C_\Psi = \int_{-\infty}^{\infty} \frac{|\hat{\Psi}(\omega)|^2}{|\omega|} d\omega < \infty ,$$

where $\hat{\Psi}(\omega)$ – Fourier transform of $\Psi(t)$.

In that case inverse CWT will exist

$$f(t) = \frac{1}{C_\Psi} \int_0^{+\infty} \int_R Wf_\Psi(a,b) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) db \frac{da}{a^2}. \qquad (3)$$

In the case of discrete signals analysis CWT expression (1) will take the form [3]:

$$W_\Psi(a,m)_\delta = \frac{1}{\sqrt{a}} \sum_{n=0}^{M-1} f[n] \cdot \psi^*\left(\frac{n-m}{a}\right),$$

where $f[n]$– samples of $f(t)$; m – shift of mother function; M – number of signal samples.

Before calculating the CWT it is necessary to select the mother wavelet and also a step and maximal time shift of scaled wavelets on that interval of argument values for which a signal is given. In the most cases of digital signal analysis the interval for which CWT will be calculated, coincides with a full signal duration interval and a step can be chosen equal to the sampling period.

In different applications the choice of step, maximal and minimum values of scaling coefficients for calculating the samples of dilated and squeezed mother wavelet in (2) is much more difficult. The number of wavelets which will be used for CWT calculations will directly depend on the value of these parameters, and also the form of wavelets and minimal and maximal duration of stretched and compressed wavelet functions. That's why the question of choice of scale coefficients deserves on particular attention.

2. General approach to scale selection

From the mathematical point of view wavelet analysis of signals (1) must be executed only for all possible values of scale from the set of real numbers, except for zero. Only in this case the inverse transform will exist. But in practice this is impossible to use these scales because of its infinitely generous amount. The task of choice of coefficients values for scaling mother wavelet-function in (2) for CWT calculations is not solved in literature yet, but the need of practical realization of CWT requires its solution to be done.

It is possible to divide the tasks of CWT use on two basic groups, first related to renewal of signal after transform, and the second is without the need to reconstruct the signal. The example of task from the first group is loss or lossless signal compression, when wavelet representations of signals are used in order to keep in memory or to transmit via communication channel not the values of signal but only coefficients of his wavelet representation. In such case the requirement emerges that the array of coefficients to occupy less memory than the signal samples itself and the initial signal's reconstruction to be possible. If mother wavelet and parameters of wavelet decomposition are selected properly, complete or partial renewal of signal is possible with (3).

Almost all tasks of signal analysis and in particular in the case of biomedical signals belong to the second group, which only studies the signal properties by using the representation of signal as the set of wavelet coefficients. The renewal of initial signal from its decomposition is not necessary. The main attention instead is given only to examination of the information, which can be obtained from exploring the coefficients of signal representation by its wavelet transform with selected mother function and for the set of scales. In this work primer attention will be given to the second group of tasks. Often the analysis of signals is used for time localization of intervals in the signals which have the preliminary known form – so-called patterns. This task will be further considered more thoroughly.

From (1) it is evident that the result of wavelet-analysis will depend on the type of mother function which is used for the decomposition. That's why it is possible to state that the use of different mother wavelets will result in different signal's representations. Thus it's possible to pick up or build mother wavelet function which is adapted to localization of patterns to be found in signal. For those scales factors of dilation or compressions at which mother function will look like pattern to be detected and have the same time and spectral properties, the values of wavelet coefficients will be greater for those shift parameters, that correspond to the time location of pattern in the signal. Consequently, analyzing time dependence of wavelet representation's coefficients for some preselected set of scales it is possible to define the areas of signal where probably there can be the pattern. A natural question appears, which scales factors need to be selected for such analysis. In this work one possible approach is developed to the decision of this question.

3. Choice of scales limit values for continuous wavelet transform

As a rule in time localization of patterns in the signal, the shape and configuration of the pattern to be detected is preliminary known beforehand. Moreover, there is often the information not only about how the template of pattern of interest looks like but also about its possible distortions in the real signals. If the minimal and maximal durations of the pattern in real signal in known, this information could be used for calculating the wavelet scale’s range, which should be employed for further analysis of wavelet coefficients. The possibility exists to calculate the preferred scales of dilations and squeezing of mother wavelet at which maximal values of decomposition coefficients should be expected in the time instants where the scaled patterns occur in the signal.

Let we have one preliminary pattern $p(t)$ of duration T_p in the signal $f(t)$, representing the example of the patterns that should be detected in the signal. These patterns could be contracted or stretched copies of pattern $p(t)$, located arbitrarily in the signal. The task of finding corresponding time instants is the task of pattern localization in the signal.

Let the minimal $T_{\min}$ and maximal $T_{\max}$ durations of possible signal parts which will be regarded as similar to the pattern of interest are known. All parts of signal $f(t)$ which have durations larger than $T_{\max}$ or smaller than $T_{\min}$ will be regarded as non-similar to the pattern of interest, and its localization will be considered as fail of the techniques.

Let for the time localization of pattern the adapted mother wavelet $\Psi(t)$ is selected or constructed with some technique and it has duration T_Ψ. Next step is the evaluation of the set of scaling coefficients for calculation the wavelet functions to further use them in (1). For using (2) the range of scales $a \in [a_{\min} ... a_{\max}]$ should be given in order to obtain the scaled wavelets and to use wavelet coefficients for localization the patterns of duration in the range $T \in [T_{\min} ... T_{\max}]$. The following technique for estimation the range of coefficient limits is proposed.

To derive the maximal coefficient $a_{\max}$ it should be noted that the corresponding scaled wavelet in the set obtained by (2) will be the one with the largest duration among all possible wavelets in the set:

$$T_{\Psi\max} = T_\Psi \cdot a_{\max}.$$

The wavelet coefficients corresponding to the scale $a_{\max}$ will have the maximal value for the shift of wavelet function where the dilated pattern $p(t)$ copy of the duration $T_{\max}$ is located in the signal. Thus the needed value of dilation coefficient $a_{\max}$ could be calculated from the equality $T_{\Psi\max} = T_{\max}$ as:

$$a_{\max} = \frac{T_{\max}}{T_\Psi}. \quad (4)$$

Similarly, the minimally dilated copy of mother wavelet $\Psi(t)$ with duration $T_{\Psi\min} = T_\Psi \cdot a_{\min}$ will correspond to minimal scaling coefficient $a_{\min}$. This duration should conform to the squeezed relative to initial $p(t)$ pattern with minimal duration $T_{\min}$ and minimal scaling coefficient for wavelet analysis could be evaluated:

$$a_{\min} = \frac{T_{\min}}{T_\Psi}. \quad (5)$$

Thus if the range of pattern’s duration in real signals is preliminary known then the proposed simple technique allow to obtain the ranges for the values of minimal and maximal dilation and squeezing coefficients for arbitrary mother wavelet function. This possibility will help in the wavelet analysis and theoretically considerably reduce the computations needed to be performed for time localization of patterns in the signal.

4. Example of selection the scaling coefficients for time localization of patterns in electroencephalogram

For experimental verification of applicability of the proposed technique it was employed in the task of time localization of epileptiform patterns in the electroencephalogram (EEG) [3].

First, the artificial EEG (Fig. 1) was simulated that contains real epileptiform complexes of different duration. With the use of previously developed method [4] mother wavelet function adapted to time localization of epileptiform complexes was created. Wavelet duration was 0.43 sec.

Taking into account, that duration of the real epileptiform complexes in electroencephalograms is in the range from 0.2 to 0.8 sec., using the formulas (4)-(5) expected limit scales needed for mother wavelet: $a_{\max} = \frac{T_{\max}}{T_\Psi} = \frac{0.8}{0.43} = 1.86$ and $a_{\min} = \frac{T_{\min}}{T_\Psi} = \frac{0.2}{0.43} = 0.47$ were calculated. The examples of wavelets scaled according to the coefficients found with the proposed technique are presented on Fig. 2.

Fig. 1. EEG signal with real epileptiform complexes

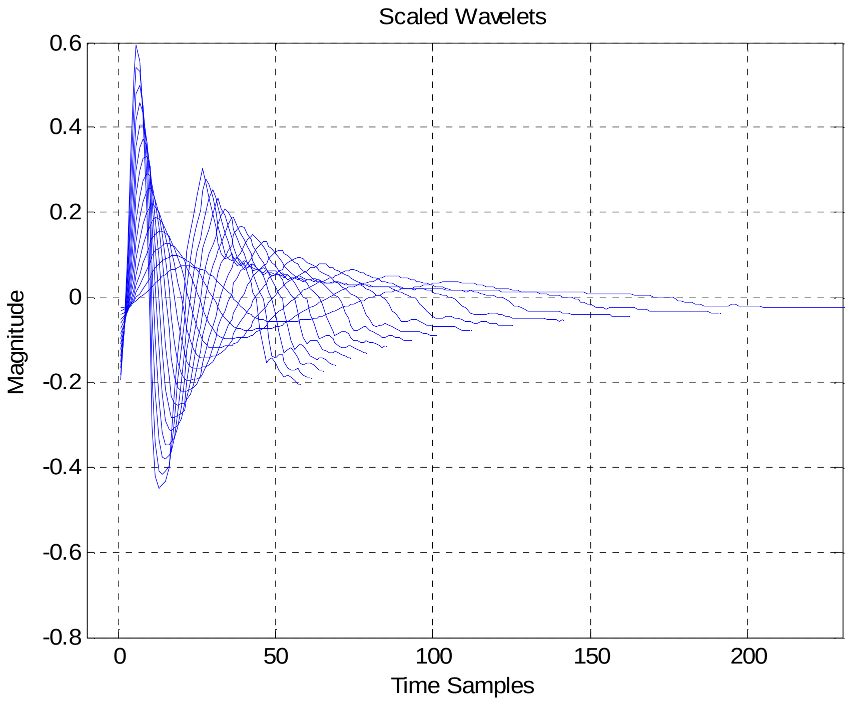

Fig. 2. Scaled adapted wavelet functions

After that the CWT computations were performed using the previously developed technique of continuous wavelet transform of discrete signals [5]. Scalogram of the EEG signal and corresponding time dependence of the closeness measure between the EEG signal and the scaled epileptiform complexes are given on Fig. 3 – 4. Peaks on Fig. 4 showed the best conformity with the epileptiform complexes. Thus preliminary selection of the scales allowed to limit the needed wavelet coefficient computation only by the range required for the epileptiform complexes localization.

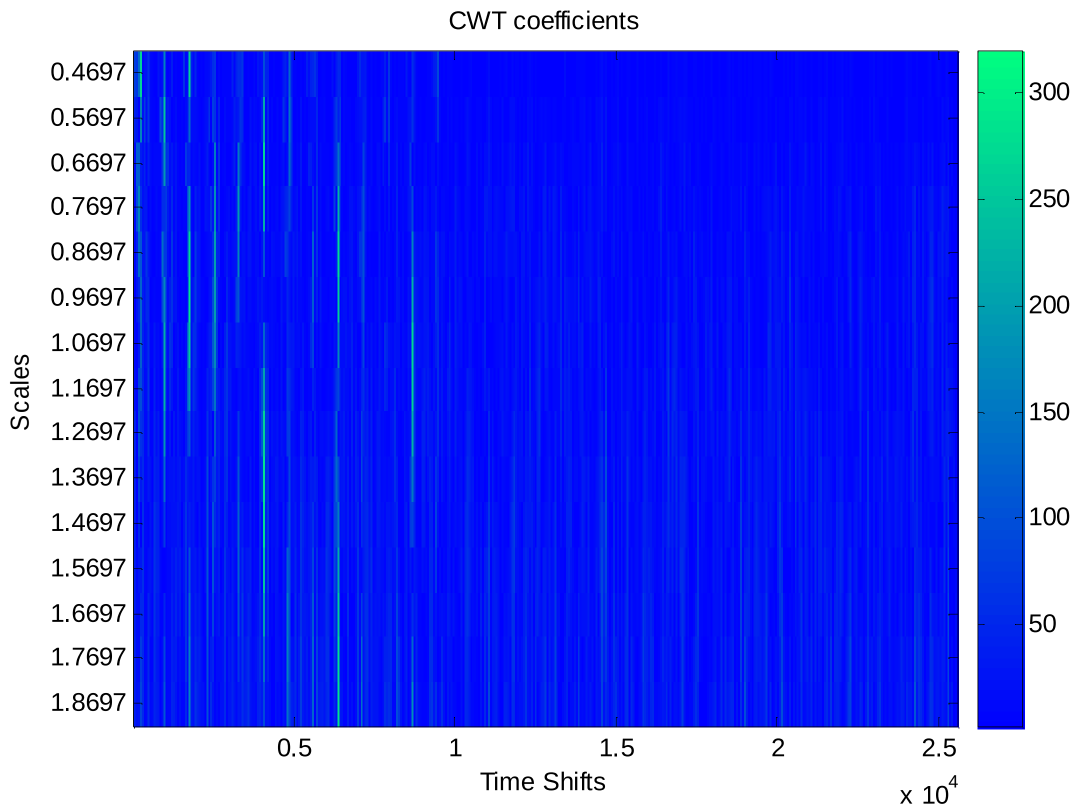

Fig. 3. EEG scalogram for the calculated range of scaling coefficient

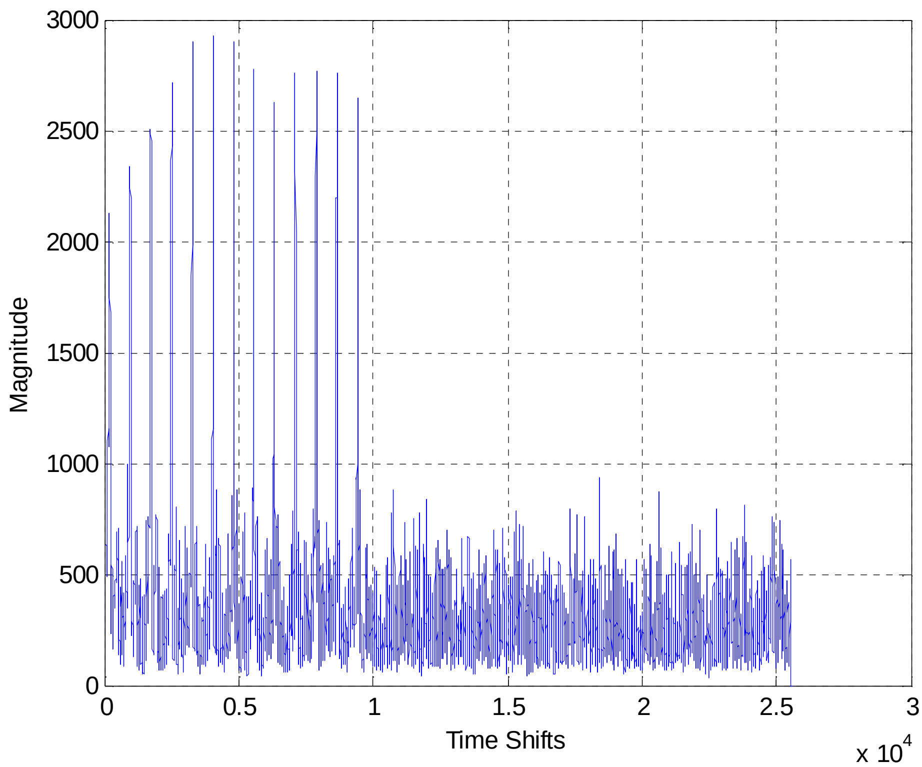

Fig. 4. Result of closeness measure calculations with wavelet coefficients

Conclusions

The new approach to selection of scales for mother wavelet function is offered. With the proposed technique the precise values of minimal and maximal scale coefficients of mother wavelet function scaling for the time localization of patterns with known duration could be calculated for given mother wavelet. Applicability of the technique is proved by successful employing it for the time localization of the epileptiform complexes in the electroencephalogram with continuous wavelet transform.

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